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Binary Weight Distributions of Low Rate Reed-Solomon Codes

Charles T. Retter

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Charles T. Retter

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U.S. Army Research Laboratory
ATTN: AMSRL-IS-TP
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13. ABSTRACT *(Maximum 200 words)*

This report summarizes the results of a study of the binary weight distributions of low rate Reed-Solomon error-correcting codes. It includes a review of the fundamental properties of Galois fields, Reed-Solomon codes, and weight distributions. Because the binary weight distribution is a good indication of the binary error-correcting capabilities of a code, computation of binary weight distributions makes it possible to select the best codes for binary channels and to estimate their true error-correcting capabilities. During the study, the weight distributions of 3,046 codes containing almost 50 trillion code words were computed. This report contains graphs of the distributions and tables of the minimum distances of all these codes. It also compares the results with previously known bounds.

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1 Introduction

This report summarizes the results of a study of the binary weight distributions of low rate Reed-Solomon codes. Although Reed-Solomon codes are among the most popular error-correcting codes in practical applications and they are very well understood, very little is known about the weight distributions of binary codes derived from them. Because the binary weight distribution is a good indication of the binary error-correcting capabilities of a code, computation of binary weight distributions makes it possible to select the best codes for binary channels and to estimate their true error-correcting capabilities.

This study was undertaken in order to find good binary codes and to improve the theoretical understanding of the relationship between the expansion being used and the properties of the resulting code. The study was restricted to binary expansions because of their practical importance, and it was restricted to codes whose rates are low enough to allow all the codewords to be examined. Since the computations were performed on a KSR1 supercomputer, it was possible to examine binary codes with dimensions as large as 42, although most of the codes in this study have dimensions between 32 and 36. All Reed-Solomon codes with parameters (31,7), (63,6), (127,5), and (255,4) were expanded using all normal bases. Then, the most promising codes with parameters (31,8), (63,7), (127,6), and (255,5) were examined. 3064 codes containing almost 50 trillion codewords were generated.

The results of the study include complete binary weight distributions for the 3064 codes. To save space, these are included in this report in the form of small graphs. However, the numerical distributions are included for the most interesting cases, and tables of minimum distances of all the codes are also included here. The minimum distances of the best codes found in this study are typically two to three times as large as the BCH bound (or Reed-Solomon d_{min}) would guarantee, and many are significantly larger than the STK bound.

Some familiarity with error-correcting codes and Galois fields is assumed. However, Section 2 reviews the basic concepts and defines the terms to be used in this report. In Section 3, the uses of weight distributions and the algorithms used to calculate them are discussed. Section 4 summarizes the results of the study, and the tables and graphs of the results appear in appendices.

2 Definitions

2.1 Galois Fields

The alphabet used in a conventional Reed-Solomon code is normally a Galois field whose size is approximately equal to the length of the code. This section reviews some of the properties of Galois fields. (See [1, 2, 3] for more details.) Since the results reported here depend on particular field elements, the representation of the elements is significant, and complete tables of the fields are included in Appendix A. Table 1 lists the fields used and the primitive polynomials in each case.

Table 1. Galois Fields

Code Length	Field	Primitive Polynomial
31	GF(32)	$x^5 + x^2 + 1$
63	GF(64)	$x^6 + x + 1$
127	GF(128)	$x^7 + x + 1$
255	GF(256)	$x^8 + x^4 + x^3 + x^2 + 1$

Using one of the above polynomials, it is easy to express all elements of the field either as powers of a primitive element or as polynomials of degree smaller than that of the primitive polynomial. For example, if α is a root of $x^5 + x^2 + 1$, then

$$\begin{array}{rcccccc} \alpha^0 & = & & & & & 1 \\ \alpha^1 & = & & & & \alpha & \\ \alpha^2 & = & & & \alpha^2 & & \\ \alpha^3 & = & & \alpha^3 & & & \\ \alpha^4 & = & \alpha^4 & & & & \\ \alpha^5 & = & & & \alpha^2 & + & 1 \\ \alpha^6 & = & & \alpha^3 & + & \alpha & \\ \ldots & = & \ldots & \ldots & \ldots & \ldots & \ldots \end{array}$$

Continuing in this way, we can construct the log table for the field GF(32), which is shown in Table A-1.

Using such a table, it is easy to do arithmetic with the field elements. To multiply two elements, we use the powers of α from the "exp" column, which are effectively logs, and simply add exponents mod 31, since $\alpha^{31} = 1$. To add two elements, we use the polynomial representation from the table and add coefficients mod 2. Naturally, the zero element must be treated separately, since it is not a power of α.

Tables A-1 through A-4 are log tables for all the Galois fields that are used here. In this report, we will express all field elements using the representation shown in the "exp" columns of the log tables. Only the exponents will be listed, so a field element will be listed as 5 rather than α^5.

To obtain a binary codeword from a Reed-Solomon codeword, each of the field elements in the Reed-Solomon codeword must be mapped into a set of bits. Although any one-to-one mapping would produce a binary code, the mapping must be linear if we want to obtain a linear binary code. The representation of field elements on the right side of the above log table could be used as a mapping, with the five binary coefficients being the binary m-tuple. In fact, this is the most popular choice, but there are many others.

Any linear mapping from GF(2^m) to binary m-tuples can be specified by a basis, which is just a list of m linearly independent field elements. If $(\delta_1, \delta_2, \ldots, \delta_m)$ is a basis, and γ is an element in GF(2^m), then the binary expansion of γ is the m-tuple $(\gamma_1, \gamma_2, \ldots, \gamma_m)$ for which

$$\gamma = \gamma_1\delta_1 + \gamma_2\delta_2 + \cdots + \gamma_m\delta_m$$

To simplify the conversion from GF(2^m) to binary m-tuples, it is most convenient to use what is called the *dual basis* $(\beta_1, \beta_2, \ldots, \beta_m)$. It is possible to calculate the β_i from the δ [1, p.110], or we can pick a dual basis directly by choosing a set of m linearly independent field elements to use as a dual basis. Using a dual basis, we can calculate the binary m-tuple corresponding to a field element γ as follows:

$$\gamma \longrightarrow (\mathrm{Tr}(\beta_1\gamma),\ \mathrm{Tr}(\beta_2\gamma),\ \ldots,\ \mathrm{Tr}(\beta_m\gamma)) \tag{1}$$

in which the trace function is defined as

$$\mathrm{Tr}(x) = \sum_{i=1}^{m} x^{2^i}$$

Since the trace of x is the sum of all its conjugates (one or more times), the value of the trace is always 0 or 1, and the mapping (1) produces a binary m-tuple. Naturally, the mapping defined in (1) can be stored in a small table, so the conversion from GF(2^m) to binary for any given basis can be done very efficiently. Tables A-1 through A-4 include the traces of all the field elements in the columns labeled 'T'.

Since the goal of this research is to evaluate the effect of the basis on the properties of the resulting binary code, we need to examine many different bases. Any linearly independent set of m field elements can serve as a basis. However, many of these produce equivalent codes. For example, expanding a Reed-Solomon code with $(\eta\beta_1, \eta\beta_2, \ldots, \eta\beta_m)$ will produce the same binary code as expanding it with $(\beta_1, \beta_2, \ldots, \beta_m)$ because multiplying one of the Reed-Solomon codewords by η will produce another codeword.

Similarly, expanding a cyclic code with $(\beta_1^2, \beta_2^2, \ldots, \beta_m^2)$ will produce a binary code with the same weight distribution as the expansion with $(\beta_1, \beta_2, \ldots, \beta_m)$. To see this, notice that

the expansion of a codeword $c(x) = c_0 + c_1x + c_2x^2 + \ldots$ with basis $(\beta_1, \beta_2, \ldots, \beta_m)$ is the same as the expansion of $c_0^2 + c_1^2x + c_2^2x^2 + \ldots$ with basis $(\beta_1^2, \beta_2^2, \ldots, \beta_m^2)$, since $\mathrm{Tr}(x) = \mathrm{Tr}(x^2)$. Assuming that the length is odd, which is normally the case for cyclic codes with characteristic 2, $c_0^2 + c_1^2x + c_2^2x^2 + \ldots$ is just a permutation of $c_0^2 + c_1^2x^2 + c_2^2x^4 + \ldots = [c(x)]^2$, which must be in the original cyclic code. So squaring the basis elements just permutes the column positions and the codewords and has no effect on the weight distribution.

Table 2. Number of Distinct Bases

Field	Number of Distinct Bases	Number of Distinct Normal Bases
GF(8)	2	1
GF(16)	16	2
GF(32)	540	3
GF(64)	74120	4
GF(128)	3.6×10^7	7
GF(256)	6.5×10^{10}	16

For the smallest fields, it is possible to evaluate all Reed-Solomon codes expanded with all possible bases. However, for the larger fields, we must choose some reasonable subset of the possible bases. Previous studies [4, 5] seem to indicate that some of the best binary codes are likely to be produced with normal bases. A normal basis has the form

$$(\beta^{2^0}, \beta^{2^1}, \ldots, \beta^{2^m})$$

in which β can be any field element for which the above powers are linearly independent. Normal bases have also been studied extensively for theoretical reasons and because they simplify the arithmetic circuitry. See Chapters 4 and 5 of [3] for more information about normal bases.

As shown in Table 2, the number of distinct normal bases is small enough to allow all cases to be examined. This report describes the weight distributions of all Reed-Solomon codes with parameters (31,7), (63,6), (127,5), and (255,4) expanded with all distinct normal bases.

Another popular method of expanding codes is to use a *polynomial basis* $(\alpha^0, \alpha^1, \ldots, \alpha^{m-1})$ in which α is the primitive element used to define the field. Weight distributions using this basis are also included in this study for comparison. Finally, in the case of GF(256), a technique for constructing a basis with unusual symmetries was described in [4]. This *r-paired* basis for GF(256) is

$$(\alpha^0, \alpha^{85}, \alpha^{51}, \alpha^{136}, \alpha^{15}, \alpha^{100}, \alpha^{66}, \alpha^{151})$$

This basis will almost always produce a *self-orthogonal* binary code. That is, a code in which the dot product of any codeword with any other codeword is zero. Reed-Solomon codes of length 255 were also expanded using this r-paired basis.

2.2 Reed-Solomon Codes

One way to define an (N, K) Reed-Solomon code over GF(2^m) is to encode the K-tuple $(I_0, I_1, I_2, \ldots, I_{K-1})$ by evaluating the Mattson-Solomon polynomial [6]

$$f_I(x) = I_{K-1}x^{K-1} + \cdots + I_2x^2 + I_1x + I_0$$

at all N of the nonzero field elements in GF(2^m). The codeword consists of the N field elements resulting from the N evaluations of the polynomial $f_I(x)$. This is equivalent to multiplying the information vector $[I_0\ I_1\ I_2 \ldots I_{K-1}]$ by the following generator matrix:

$$G = \begin{bmatrix} \alpha^0 & \alpha^0 & \alpha^0 & \cdots & \alpha^{0(N-1)} \\ \alpha^0 & \alpha^1 & \alpha^2 & \cdots & \alpha^{1(N-1)} \\ \alpha^0 & \alpha^2 & \alpha^4 & \cdots & \alpha^{2(N-1)} \\ \alpha^0 & \alpha^3 & \alpha^6 & \cdots & \alpha^{3(N-1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha^0 & \alpha^{(K-1)} & \alpha^{2(K-1)} & \cdots & \alpha^{(N-1)(K-1)} \end{bmatrix}$$

Expressing the code in terms of polynomial evaluation allows us to use the fundamental theorem of algebra to bound the minimum weight of the code. Since a polynomial of degree $(K-1)$ can have at most $(K-1)$ zeros, every nonzero codeword must have at least $N-(K-1)$ nonzero symbols. So the minimum weight of a Reed-Solomon code is at least $N+1-K$. This is a special case of the *BCH bound* on cyclic codes. Furthermore, since K symbols can be chosen as information symbols (using a systematic encoder), there must be some codewords with $(K-1)$ zeros and weight exactly $N+1-K$. So the minimum distance of an (N, K) Reed-Solomon code is exactly $N+1-K$.

Another way of looking at this encoding process is to view each row of the G matrix as having a different *frequency*. The encoding process, multiplication by the above matrix, is described by the equations

$$C_i = \sum_{j=0}^{K-1} I_j\alpha^{ij} \qquad i = 0, \ldots, N-1$$

Since α is a primitive N-th root of unity, this equation has exactly the same form as a Discrete Fourier Transform, with $I_k, \ldots, I_{N-1}$ equal to zero. So we can think of the codeword as a signal whose DFT is confined to frequencies 0 to $(K-1)$. We will call the band of frequencies that may have nonzero coefficients the *spectrum* of the code. Viewed this way, it is possible to think of the decoding process as a kind of digital filtering.

To enlarge the set of possible codes, we can allow the codewords to be bandlimited within any contiguous band of K frequencies. That corresponds to evaluating a polynomial such as

$$f_I(x) = I_{K-1}x^{s+K-1} + \cdots + I_2x^{s+2} + I_1x^{s+1} + I_0x^s$$

This polynomial has degree $(s+K-1)$, but s of its roots are at 0, and we are not evaluating it at 0, so the minimum distance is still $N+1-K$. Using a different starting frequency makes no difference in the weights of the Reed-Solomon code, but it can make a big difference in the binary expansion. The generator matrix for this version of a Reed-Solomon code looks like this:

$$G = \begin{bmatrix} \alpha^0 & \alpha^s & \alpha^{2s} & \cdots & \alpha^{(N-1)s} \\ \alpha^0 & \alpha^{s+1} & \alpha^{2(s+1)} & \cdots & \alpha^{(N-1)(s+1)} \\ \alpha^0 & \alpha^{s+2} & \alpha^{2(s+2)} & \cdots & \alpha^{(N-1)(s+2)} \\ \alpha^0 & \alpha^{s+3} & \alpha^{2(s+3)} & \cdots & \alpha^{(N-1)(s+3)} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \alpha^0 & \alpha^{(s+K-1)} & \alpha^{2(s+K-1)} & \cdots & \alpha^{(N-1)(s+K-1)} \end{bmatrix} \tag{2}$$

The binary codes whose weight distributions were evaluated in this study consist of Reed-Solomon codes generated by the G matrix (2) with the symbols expanded using a dual basis as shown in (1). To form a binary generator matrix, we can expand each row of (2) using the dual basis (1), but this would produce a K by mN binary matrix. Since the information vector will be binary, we need an mK by mN matrix, so we must expand m different linearly independent multiples of each row of (2). The particular multiples of rows that we choose will have no effect on the weight distribution of the binary code, only on the mapping from information vectors to codewords. However, the multiples must be linearly independent, so it is convenient to use the same basis elements as (1). When each element in (2) is replaced by an m by m binary matrix, the resulting generator matrix looks like this:

$$G = \begin{bmatrix}
\left\{\begin{matrix} \mathrm{Tr}(\beta_1\beta_1) & \mathrm{Tr}(\beta_1\beta_2) & \cdots \\ \mathrm{Tr}(\beta_2\beta_1) & \mathrm{Tr}(\beta_2\beta_2) & \cdots \\ \vdots & \vdots & \vdots \\ \mathrm{Tr}(\beta_m\beta_1) & \mathrm{Tr}(\beta_m\beta_2) & \cdots \end{matrix}\right\} & \left\{\begin{matrix} \mathrm{Tr}(\beta_1\beta_1\alpha^s) & \mathrm{Tr}(\beta_1\beta_2\alpha^s) & \cdots \\ \mathrm{Tr}(\beta_2\beta_1\alpha^s) & \mathrm{Tr}(\beta_2\beta_2\alpha^s) & \cdots \\ \vdots & \vdots & \vdots \\ \mathrm{Tr}(\beta_m\beta_1\alpha^s) & \mathrm{Tr}(\beta_m\beta_2\alpha^s) & \cdots \end{matrix}\right\} & \begin{matrix} \cdots \\ \cdots \\ \cdots \end{matrix} \\
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\vdots & \vdots & \vdots \\
\left\{\begin{matrix} \mathrm{Tr}(\beta_1\beta_1) & \mathrm{Tr}(\beta_1\beta_2) & \cdots \\ \mathrm{Tr}(\beta_2\beta_1) & \mathrm{Tr}(\beta_2\beta_2) & \cdots \\ \vdots & \vdots & \vdots \\ \mathrm{Tr}(\beta_m\beta_1) & \mathrm{Tr}(\beta_m\beta_2) & \cdots \end{matrix}\right\} & \left\{\begin{matrix} \mathrm{Tr}(\beta_1\beta_1\alpha^{s+K-1}) & \mathrm{Tr}(\beta_1\beta_2\alpha^{s+K-1}) & \cdots \\ \mathrm{Tr}(\beta_2\beta_1\alpha^{s+K-1}) & \mathrm{Tr}(\beta_2\beta_2\alpha^{s+K-1}) & \cdots \\ \vdots & \vdots & \vdots \\ \mathrm{Tr}(\beta_m\beta_1\alpha^{s+K-1}) & \mathrm{Tr}(\beta_m\beta_2\alpha^{s+K-1}) & \cdots \end{matrix}\right\} & \begin{matrix} \cdots \\ \cdots \\ \cdots \end{matrix}
\end{bmatrix}$$

By choosing different starting frequencies for the Reed-Solomon code and different bases

for the expansion, a large number of different binary codes can be obtained. As the tables and graphs in this report will show, the characteristics of these binary codes vary greatly.

The BCH bound, which specifies that $d_{min} \geq N+1-K$, applies to all these codes but is usually a very weak bound for expansions of low rate Reed-Solomon codes. A better bound has been published by Sakakibara, Tokiwa, and Kasahara [7]. This bound views the expansion of a codeword on each coordinate as a word in a binary cyclic code, and bounds the minimum weight of the complete expansion as the smallest product of the weight of each such binary cyclic codeword and the number of coordinates that must be nonzero for such a codeword to be present. This *STK bound* has been computed for the cases covered in this study, and the tables in Appendix B show that it is considerably tighter than the BCH bound.

3 Weight Distributions

3.1 Uses of Weight Distributions

The weight distribution of a linear code is useful because it gives a very good indication of the performance of the code on channels in which the errors are independent of each other. For example, suppose that A_i is the number of codewords with weight i in a binary linear (n,k) code. On a binary symmetric channel where each bit has a probability q of being received correctly and a probability $p = (1-q)$ of being received incorrectly, the probability of an error being detected by this code is

$$P_d = 1 - \sum_{i=1}^{n} A_i \, p^i \, q^{n-i}$$

since undetected errors occur only when the error pattern is exactly equal to another codeword.

When p is very small (the channel has high signal-to-noise ratio), the expression for P_d is dominated by the first nonzero term of the summation, the one in which i is equal to the minimum distance of the code. The minimum distance is also a useful parameter because the code can guarantee to correct all error patterns with $\lfloor d_{min}/2 \rfloor$ or fewer errors. However, on channels with lower signal-to-noise ratios, those with $p^i \approx p^{i+1}$, error patterns of larger weights are still quite likely, and sometimes it is possible for a code to correct most of the error patterns of weights considerably larger than $\lfloor d_{min}/2 \rfloor$.

The best possible decoder for a code is called a *maximum likelihood* decoder, because it finds the codeword which is most likely to have been transmitted given the received pattern. Using the weight distribution, reasonably tight bounds on the performance of a maximum

likelihood decoder can be obtained [8, 9, 10]. It is easy to calculate the probability of any particular error pattern for a binary symmetric channel; if the pattern has weight i, the probability is $p^i q^{n-i}$. Such a pattern will be decoded incorrectly by a maximum likelihood decoder if it is closer to another codeword than to the all-zero codeword. If we multiply this probability by the number of error patterns of weight i that are closer to the codeword than to the all-zero codeword, and then we sum over all of the codewords, we obtain a simple upper bound on the probability of decoding error. Unfortunately, this bound is tight only for small p. For noisy channels, we must account for the fact that many error patterns are closer to multiple codewords than to the all-zero codeword.

Poltyrev [10] has recently published a bound which is tight for larger values of p. His bound can be expressed as

$$P_e \leq \sum_{w=dmin}^{2(m_0-1)} A_w \Gamma_w + \sum_{i=m_0}^{n} \binom{n}{i} p^i q^{n-i} \tag{3}$$

in which the coefficients are given by

$$\Gamma_w = \sum_{i=\lceil w/2 \rceil}^{m_0-1} \binom{w}{i} p^i q^{w-i} \sum_{j=0}^{m_0-i-1} \binom{n-w}{j} p^j q^{n-w-j} \tag{4}$$

and m_0 is the smallest integer m for which

$$\sum_{w=dmin}^{2m} A_w \sum_{i=\lceil w/2 \rceil}^{m} \binom{w}{i} \binom{n-w}{m-i} \geq \binom{n}{m}$$

In practice, m_0 does not vary much for codes with the same n and k, so we can calculate the coefficients Γ_w and use them to compare the weight distributions of various codes. Figure 1 shows the values of the coefficients Γ_w for (2040,32) binary codes at some relatively high channel error probabilities. By using these coefficients, we can easily calculate the upper bound on decoded error probability for any code, given its weight distribution. In fact, we can also see which terms in the weight distribution contribute the most to the decoded error probability. In many cases, the most important term is the minimum distance one, but for high channel error probabilities, this is not always the case, as we will see in the next section.

It is often easier to prove results about the set of all possible codes than to prove the same results about a specific code. For example, the weight distribution of a randomly chosen code can be approximated by the binomial distribution:

$$A_w = \binom{n}{w} 2^{k-n} \approx 2^{nH(w/n)+k-n}$$

To estimate the performance of such a code, we can substitute this in equation (3) and obtain the decoded error probabilities shown in Figure 2 for (2040,32) codes. Similarly, we could

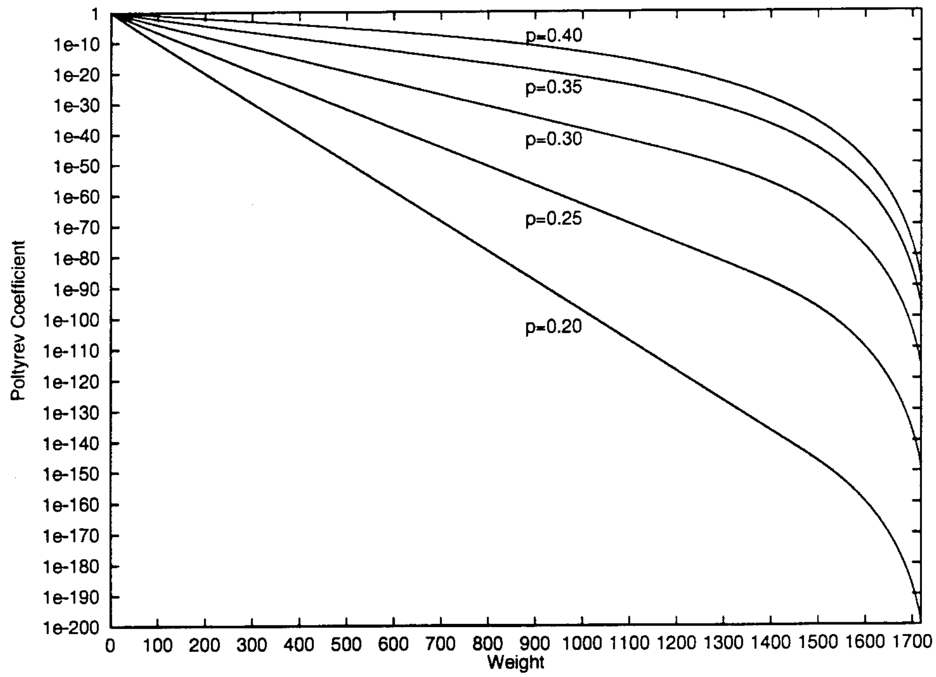

Figure 1. Coefficients Γ_w in the Poltyrev Bound (Equation 4)

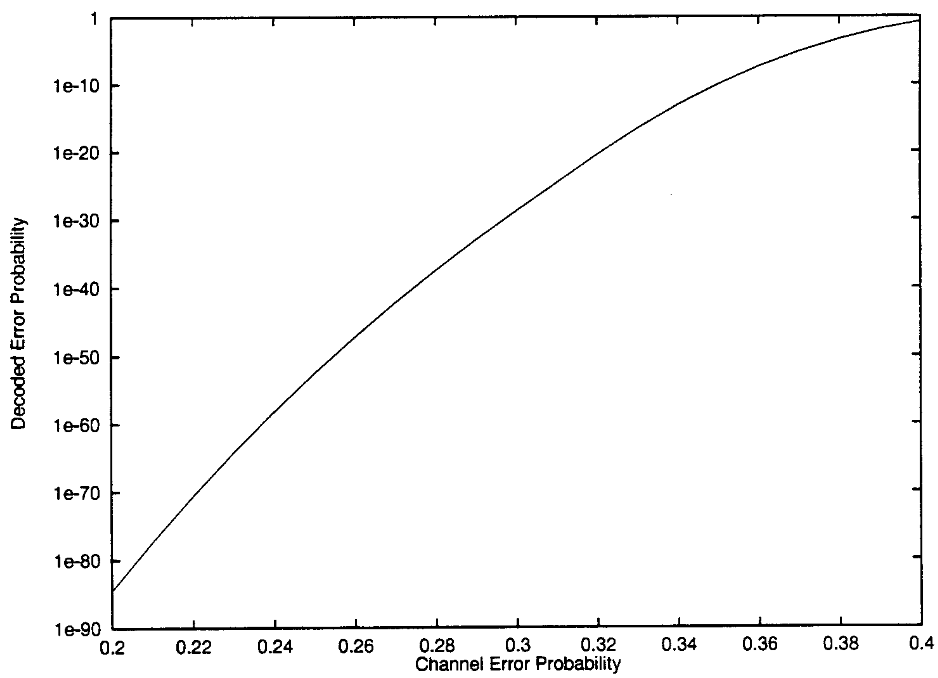

Figure 2. Poltyrev Bound for a (2040,32) Binomial Weight Distribution

use the average weight distribution for GRS codes given in [11]. producing a curve slightly better than that shown in Figure 2 for small p. Either of these averages can be used as a reference in evaluating particular codes.

Weight distributions can also be used to provide information about the dual of a code. The MacWilliams identities [12] make it relatively easy to find the weight distribution of the dual from the weight distribution of the original code. If there are A_i words of weight i in a binary linear (n,k) code, then the number of words of weight j in the $(n, n-k)$ dual code is

$$\begin{aligned} B_j &= 2^{-k} \sum_{i=0}^{n} P_j(i) \\ &= 2^{-k} \sum_{i=0}^{n} \sum_{h=0}^{j} (-1)^h \binom{i}{h} \binom{n-i}{j-h} \end{aligned}$$

in which the $P_j(x)$ are called *Krawtchouk polynomials.* See Chapter 5 of [12] for more information about Krawtchouk polynomials and ways to calculate the dual weight distribution. Using the weight distributions found in this study, it is relatively easy to calculate the weight distributions of the dual codes, even though the number of codewords in any of the dual codes is huge.

3.2 Calculation of Weight Distributions

In some cases, it is possible to determine the weight distribution of a code by reasoning about its algebraic properties. For example, the weight distribution of an $(N = 2^m - 1, K)$ Reed-Solomon code over GF(2^m) can be shown to be [12, p.321]

$$A_i = N\binom{N}{i} \sum_{j=0}^{i-N+K-1} (-1)^j \binom{i-1}{j} 2^{m(i-N+K-1-j)}$$

However, this A_i is the number of codewords containing i nonzero *symbols.* If we map each symbol into a binary m-tuple, the number of nonzero bits in the codeword could be any value between i and mi.

The most direct way to obtain the weight distribution, for reasonably small codes, is simply to generate all the codewords and count the number of nonzero symbols in each. A collection of programs was written to do this for binary mappings of Reed-Solomon codes. To make them as general as possible, one program produces a binary generator matrix when given the spectrum of the Reed-Solomon code and a list of the dual basis elements. The other programs calculate the weight distribution for any binary linear code, given the generator matrix. This allows them to be used with other binary codes that have less structure than those described here.

Although it is relatively easy to form the generator matrix for one of these binary codes, an (N, K) Reed-Solomon code will produce an (mN, mK) binary code, which contains 2^{mK} codewords of mN bits each. For example, a (127,6) Reed-Solomon code over GF(128) will produce an (889,42) binary code which contains 2^{42} codewords of 889 bits each. Generating all 4398046511104 of these codewords and counting the number of 1's in each of them involves a large amount of computation. In fact, only one code of this size was evaluated during this study.

Because the amount of computation is so large, the programs that calculate the weight distributions have been optimized very carefully. Since all linear combinations of the rows of the generator matrix are codewords, the programs generate codewords by choosing a row and XORing it with the previous codeword. By choosing rows using a Gray code, we can generate all the codewords with only a single XOR operation for each.

Finding the weight of a codeword is somewhat more difficult. Some computers have instructions that count the number of 1s in a word. For other machines, various algorithms can be used. The most obvious approach is simply to examine each bit and count the 1s. However, there are a number of algorithms for counting bits that are significantly faster. For example, this operation removes the least significant 1 from x:

```
x &= x-1;
```

So repeating it until x is zero counts the bits somewhat faster if there are not very many 1s in the word.

Another approach is to break the codeword into bytes (or larger pieces) and to use a table to determine the weight of each byte. On a machine with a large cache and fast load instructions, that can be very efficient.

Other algorithms use arithmetic operations to count more than one bit at a time. For example, if the machine has a fast shift operation and x is a 64-bit variable,

```
x = (0x5555555555555555 & x>>1)  + (0x5555555555555555 & x);
x = (0x3333333333333333 & x>>2)  + (0x3333333333333333 & x);
x = (0x0f0f0f0f0f0f0f0f & x>>4)  + (0x0f0f0f0f0f0f0f0f & x);
x = (0x00ff00ff00ff00ff & x>>8)  + (0x00ff00ff00ff00ff & x);
x = (0x0000ffff0000ffff & x>>16) + (0x0000ffff0000ffff & x);
x = (x>>32) + (0x00000000ffffffff & x);
```

This adds each pair of bits, leaving the sum in a 2-bit field. Then, each pair of 2-bit numbers is added, followed by pairs of 4-bit numbers, etc. If the machine has a fast mod operation, we can improve the algorithm this way:

```
x = (0x5555555555555555 & x>>1)  + (0x5555555555555555 & x);
x = (0x3333333333333333 & x>>2)  + (0x3333333333333333 & x);
x = (0x0f0f0f0f0f0f0f0f & x>>4)  + (0x0f0f0f0f0f0f0f0f & x);
x = x % 255;
```

After the first three steps, x consists of 8 bytes, each of which holds a number between 0 and 8. The mod operation adds these 8 bytes together, producing the final count.

If the codeword is too large to fit in a single 64-bit register, it may not be necessary to repeat the entire algorithm for each 64-bit section of the codeword, because many of the fields shown above are large enough to hold bit-counts from more than two of the previous fields. After the first few steps are done on a 64-bit section of the codeword, the result can be combined with the corresponding result from another 64-bit section, so the remaining steps are done only once.

The choice of the best bit-counting algorithm depends on the characteristics of the machine being used. The computations described here were done on a 256-processor KSR1. The processors on this machine are 64-bit RISCs, which issue two instructions per clock cycle. The scheduling of instructions to be issued together is determined by the compiler, with the restriction that one must be some kind of arithmetic operation and the other must be a load, store, or address computation. If the appropriate type of instruction cannot be executed during a given cycle, a no-op is inserted instead. The KSR1 has an unusual memory system, which allows any processor to use the memory of other processors essentially as virtual memory. However, for a small program which requires as much speed as possible, the most important factor is that the KSR1 has relatively long delays when cache misses occur. For that reason, keeping all data within the 256-kb caches is very helpful in maximizing execution speed.

The KSR1 has fast shifting and addition instructions but no integer division or mod instructions, and its memory load instruction is somewhat slow, especially if tables are used that are too large for the 256kb caches. Thirteen bit-counting algorithms were tested on the KSR1. The fastest used an arithmetic approach similar to the masking algorithm described above, interleaved with the memory accesses that are required to generate the codeword. Since no large tables were required for this algorithm, it was relatively easy to keep all the data within the cache. The inner loops containing the bit-counting algorithm were completely unrolled, and the arithmetic and memory access operations were interleaved manually, since the compiler's optimizer was not able to do this very well by itself. Different programs were created for various codeword lengths, with slightly different bit-counting algorithms being used in the unrolled inner loops. The resulting programs process about five bits per clock cycle. That is, a 2048-bit codeword can be generated and its weight determined in approximately 410 clock cycles. Using 64 processors, a typical code included in this study can be evaluated in less than an hour.

3.3 Choice of Codes

To evaluate a large number of promising codes, it was necessary to restrict the size of most of the codes to about 2^{35} codewords. With codes of this size, it was possible to examine all combinations of spectra using the most promising bases. Starting with spectra centered at 0, all frequencies up to N/2 were used. The spectra past N/2 produce codes that are equivalent to those below N/2. As described in Section 2.1, the most promising bases seem to be normal bases, so all normal bases were used. In addition, the popular polynomial basis was used, and the r-paired basis for GF(256) was also used. The following combinations were evaluated:

Table 3. Summary of Codes Evaluated

Reed-Solomon Parameters	Binary Parameters	Distinct Spectra	Distinct Bases	Total Codes
(31,7)	(155,35)	16	4	64
(63,6)	(378,36)	32	5	180
(127,5)	(889,35)	64	8	512
(255,4)	(2040,32)	128	18	2304

After all these codes had been evaluated, one large code of each length was chosen by looking for pairs of adjacent frequency spectra that produced codes with large minimum distances. The codes chosen were

Table 4. Large Codes Evaluated

Reed-Solomon Parameters	Binary Parameters	Spectrum	Basis
(31,8)	(155,40)	1-8	0 3 9 14 21
(63,7)	(378,42)	8-14	23 46 29 58 53 43
(127,6)	(889,42)	6-11	21 42 84 41 82 37 74
(255,5)	(2040,40)	67-71	5 10 20 40 80 160 65 130

4 Summary of Results

4.1 Minimum Distances

As explained in Section 3.1, the minimum distance of a code is a good measure of its error-correcting capability on a channel where the errors are independent and not too frequent.

The minimum distances of all the (155,35), (378,26), (889,35) and (2040,32) codes are listed in Appendix B. The STK bound is also listed for each spectrum. In some cases, the STK bound is equal to the computed minimum distance of one or more codes, so the bound is as tight as possible. However, in other cases there is a significant difference between the STK bound and the worst code examined in this study. In those cases, it is not clear whether the STK bound is loose or the particular codes examined happened to be good.

Binary expansions of Reed-Solomon codes whose spectra include frequency 0 always contain codewords of weight N, so their minimum distances are close to the BCH bound ($d_{min} \geq N+1-K$) if K is small. However, if we restrict ourselves to codes without frequency 0 in the spectrum and expansions with normal bases, the minimum distances of all the codes examined in this study were much greater than the BCH bound. The minimum distances of these codes are summarized in the following table.

Table 5. Summary of Minimum Distances

Reed-Solomon Parameters	Binary Parameters	Worst d_{min}	Average d_{min}	Best d_{min}	BCH Bound
(31,7)	(155,35)	40	40.944	44	25
(63,6)	(378,36)	84	123.690	136	58
(127,5)	(889,35)	320	359.405	368	123
(255,4)	(2040,32)	680	863.402	920	252

For comparison, the parameters of binary BCH codes with comparable lengths and rates have been calculated. As can be seen from Table 6, the best binary mappings of low rate Reed-Solomon codes should have error-correction capabilities similar to BCH codes with comparable lengths and rates, assuming that bounded distance decoders are used in both cases. Whether BCH or RS codes are more useful in a given application will depend on the decoders being used, which is beyond the scope of this report.

Table 6. Summary of Best Codes Found

Best Reed-Solomon			Comparable BCH Code		
(n,k,d)	Rate	d/n	(n,k,d)	Rate	d/n
(155,35,44)	0.226	0.284	(255,55,63)	0.216	0.247
(378,36,136)	0.095	0.360	(511,49,187)	0.096	0.366
(889,35,368)	0.039	0.414	(1023,46,439)	0.045	0.429
(2040,32,920)	0.016	0.451	(2047,34,959)	0.017	0.468

Finally, four large codes were examined by choosing spectra and bases that seemed most promising from the minimum distances of the smaller codes. Their minimum distances were not quite so close to the comparable BCH codes, but better choices of spectra and bases may well exist. Tables 7 through 10 show the complete weight-distributions of these codes.

Table 7. Weight Distribution of a (155,40) Binary Code

Spectrum 1-8, Dual Basis (0 3 9 14 21)

wt	count	wt	count
32	310	80	259753247248
40	1240	84	163600049605
44	89280	88	68179650980
48	3039860	92	18699672905
52	57662635	96	3353454140
56	695707580	100	389194057
60	5363346115	104	28871540
64	26885429365	108	1346485
68	88221337755	112	39060
72	190890926420	116	620
76	273388560575		

Table 8. Weight Distribution of a (378,42) Binary Code

Spectrum 8-14, Dual Basis (23 46 29 58 53 43)

wt	count	wt	count	wt	count
128	1512	172	156687672834	216	15201292071
132	16884	176	295524302661	220	4423925457
136	226044	180	470472479673	224	1081650780
140	1979208	184	632297829429	228	222598026
144	14846727	188	717672073860	232	38368701
148	94314969	192	688032904356	236	5602905
152	502220628	196	557065475091	240	681786
156	2236624992	200	380866179141	244	71442
160	8379432747	204	219784303809	248	3780
164	26407187163	208	107031655206	252	807
168	70054396110	212	43946192304		

Table 9. Weight Distribution of an (889,42) Binary Code

Spectrum 15-20, Dual Basis (21 42 84 41 82 37 74)

wt	count	wt	count	wt	count	wt	count
352	889	400	5539655037	448	464998845464	496	1217007218
356	9779	404	11555952758	452	408293040918	500	448900550
360	48895	408	23864037743	456	355051321646	504	163371149
364	209804	412	43074558758	460	269989677825	508	52263421
368	813435	416	76928295724	464	203322063637	512	16343376
372	3222625	420	120164224238	468	133863641086	516	4493895
376	12128627	424	185841098345	472	87275195147	520	1220597
380	39155116	428	251223266539	476	49739779108	524	306705
384	124843159	432	336422150624	480	28061294779	528	72898
388	347626559	436	393895228083	484	13839741307	532	17018
392	962951719	440	456689102114	488	6756910286	536	3556
396	2320725610	444	463060155285	492	2881537925	560	127

Table 10. Weight Distribution of a (2040,40) Binary Code

Spectrum 67-71, Dual Basis (5 10 20 40 80 160 65 130)

wt	count	wt	count	wt	count	wt	count
884	4080	956	1400404920	1028	72958062240	1100	145554000
888	7140	960	2280545614	1032	67481123280	1104	76667790
892	14280	964	3588543600	1036	60456320040	1108	39498480
896	12240	968	5485726260	1040	52489835460	1112	19354500
900	44880	972	8119459080	1044	44162913480	1116	9214680
904	128520	976	11644222590	1048	36045279180	1120	4219485
908	361080	980	16183903440	1052	28458275400	1124	1938000
912	887400	984	21806623860	1056	21815270570	1128	817020
916	1864560	988	28469815680	1060	16189517520	1132	342720
920	4168740	992	36043684410	1064	11642600280	1136	139740
924	9345240	996	44171436600	1068	8122986240	1140	53040
928	19663815	1000	52506293160	1072	5484394650	1144	33660
932	38445840	1004	60453329400	1076	3587048280	1148	6120
936	76176660	1008	67464283080	1080	2278098600	1152	2295
940	146378160	1012	72940832400	1084	1400588520	1280	51
944	268636890	1016	76471572600	1088	835191435		
948	479212320	1020	77664997080	1092	479867160		
952	834384600	1024	76483103700	1096	267899940		

4.2 Error Probability for Maximum Likelihood Decoders

Section 3.1 described Poltyrev's bound on the probability of error using a maximum likelihood decoder in terms of the weight distribution of the code. Since this bound is reasonably tight, it allows us to estimate the performance that could be expected from a code even on very noisy channels. We have computed the Poltyrev coefficients Γ_w, defined in Equation 4, for a variety of noisy channels. Using these coefficients, the Poltyrev bound on decoded error probability was computed for each code in this study. The results were compared with each other and with the bounds for randomly chosen codes.

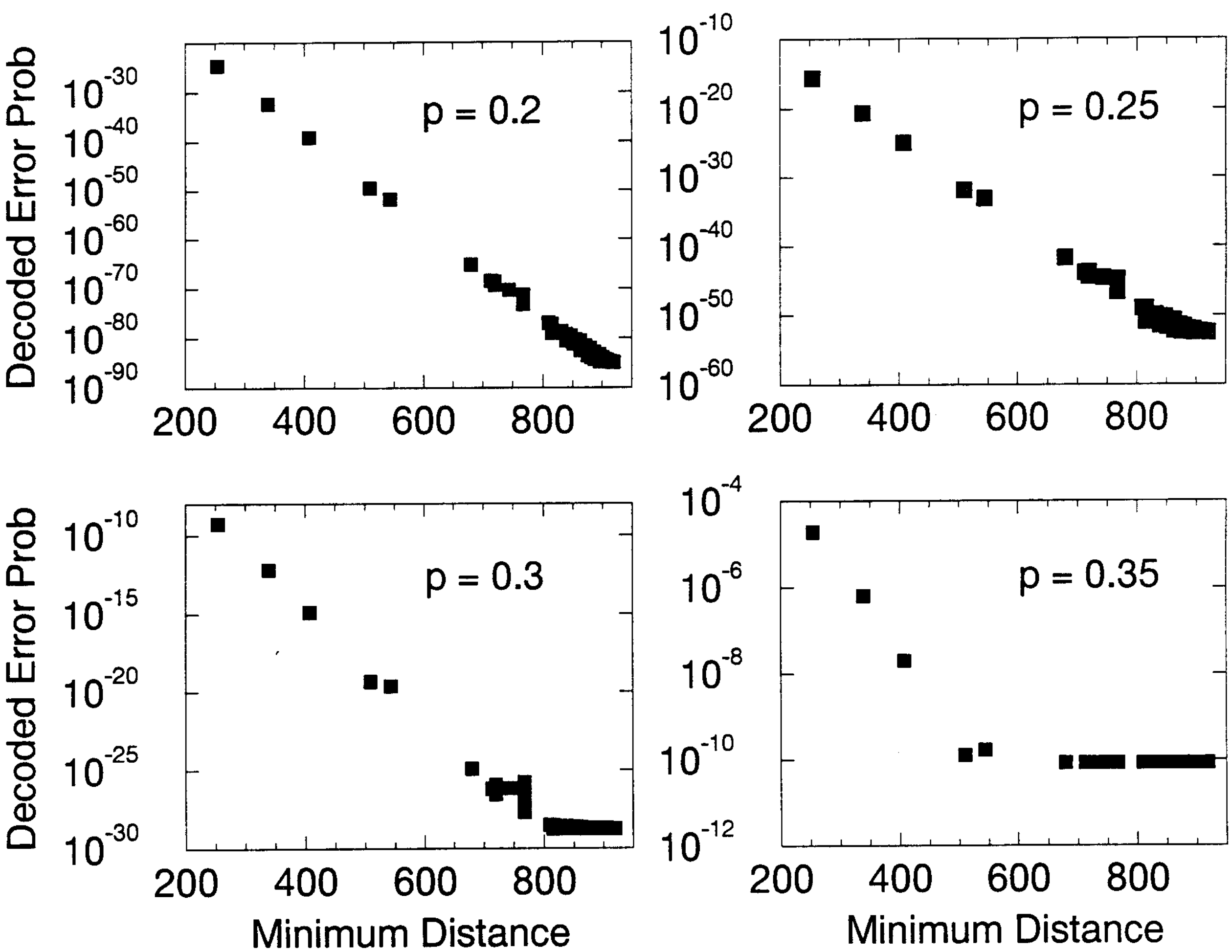

Figure 3. Poltyrev Bound versus Minimum Distance for (2040,32) Codes

Figure 3 shows the Poltyrev bounds for all the (2040,32) codes over several noisy channels. To explore the relationship between the minimum distance of a code and its expected performance, these graphs show the Poltyrev bound versus minimum distance. In general,

Table 11. Poltyrev Bounds for Randomly Chosen Codes and the Best Codes Found

(155,35) Codes						
p	Binomial Bound	GRS Bound	Best Code Found			
			Spectrum	Basis	Bound	d_{min}
0.001	7.306639e-33	7.464097e-48	1-7	p	6.206917e-51	44
0.005	2.996015e-28	7.806341e-34	1-7	p	1.791746e-35	44
0.010	5.745637e-25	9.361781e-28	1-7	p	9.168231e-29	44
0.050	9.640729e-13	8.522276e-13	1-7	n1	9.716880e-13	44
0.100	2.370380e-05	2.365799e-05	30-5	p	2.380773e-05	31
0.150	5.024232e-02	5.022447e-02	30-5	p	5.024455e-02	31

(378,36) Codes						
p	Binomial Bound	GRS Bound	Best Code Found			
			Spectrum	Basis	Bound	d_{min}
0.01	2.058723e-74	2.123295e-89	6-11	n1	2.496888e-93	136
0.05	7.036118e-45	6.149502e-46	6-11	n1	9.690504e-47	136
0.10	4.267098e-27	3.859954e-27	6-11	n1	4.387516e-27	136
0.15	4.600400e-15	4.595619e-15	6-11	n1	4.893753e-15	136
0.20	5.770604e-07	5.770402e-07	21-26	n3	5.812251e-07	136
0.25	1.665089e-02	1.665078e-02	21-26	n3	1.668319e-02	136

(889,35) Codes						
p	Binomial Bound	GRS Bound	Best Code Found			
			Spectrum	Basis	Bound	d_{min}
0.10	4.191286e-77	4.689137e-78	42-46	n5	4.864483e-79	368
0.15	1.839778e-50	1.620368e-50	42-46	n5	1.369348e-50	368
0.20	1.537811e-31	1.527868e-31	49-53	n6	1.938512e-31	368
0.25	1.672228e-16	1.672156e-16	15-19	n3	1.691448e-16	368
0.30	2.545667e-06	2.545663e-06	15-19	n3	2.547159e-06	368
0.35	2.758823e-01	2.758823e-01	15-19	n3	2.759070e-01	368

(2040,32) Codes						
p	Binomial Bound	GRS Bound	Best Code Found			
			Spectrum	Basis	Bound	d_{min}
0.20	2.720149e-85	2.142967e-85	11-14	n2	1.039228e-85	920
0.25	2.591987e-53	2.563703e-53	11-14	n2	2.909820e-53	920
0.30	1.536953e-29	1.535678e-29	95-98	n12	1.817308e-29	912
0.35	8.340791e-11	8.340779e-11	125-128	n7	8.356173e-11	896
0.40	1.559268e-01	1.559268e-01	125-128	n7	1.559512e-01	896

there was a strong correlation between the minimum distance and the bound, even for very noisy channels. The worst codes were almost always those with small minimum distances, which in this case means codes whose spectrum includes frequency 0. Conversely, codes with very large minimum distances always produced very good bounds.

However, with noisy channels, the bound exhibits a threshold effect. Any code whose minimum distance exceeds the threshold will have a very good decoded error probability, while codes below the threshold become worse as their minimum distances decrease. Although it is not obvious from the graphs, 2005 of the 2305 codes have minimum distances exceeding 800, so most of the codes perform very well on noisy channels. When the minimum distance is near the threshold, the bound varies greatly, depending on the number of codewords at or near d_{min}. This is most obvious in the graph for $p = 0.3$, where the best code with $d_{min} = 768$ has only 85 codewords of weight 768, while the worst has 7140 codewords of that weight and its bound is worse by a factor of 77.

The Poltyrev bound was also calculated for randomly chosen codes (based on the binomial distribution), and for randomly chosen binary mappings of GRS codes (based on the distribution in [11]). Some of the results are shown in Table 11. When the channel error probability is small, the best codes perform significantly better than randomly chosen codes, which could be predicted simply from the values of d_{min}. On noisy channels, the best codes found in this study are slightly worse than the average binomial or GRS families. However, the bounds for all the codes examined were very close on noisy channels, so codes with the largest d_{min} still perform very well in these cases. Since the actual channel error probability is likely to vary, codes with the largest minimum distances seem to be the best choice when either bounded distance or maximum likelihood decoders are used. However, this is not necessarily true for all other decoders, including some that we are investigating.

4.3 Gaps in the Weight Distributions

The weight distributions of almost all codes resemble the normal distribution. When the minimum distance of the dual code is large, Sidelnikov [13, 14] showed that the cumulative weight distribution differs from the cumulative normal distribution by at most $9/\sqrt{d^{\perp}_{min}}$. This was improved somewhat by Kasami et al [15]. The codes in this study have small values of $d^{\perp}_{min}$, so this bound becomes trivial, but their weight distributions are clearly close to the normal or binomial distributions. The most obvious difference is that almost all the weight distributions in this study contain regular gaps, weights for which there are no codewords.

These gaps were previously investigated in [5]. In most cases, all the weights in a code are multiples of some power of 2. A lower bound on this power of 2 can be obtained by examining the frequencies in the spectrum of the Reed-Solomon code. However, expansions with some bases result in larger gaps than expansions with other bases. In [5], this was explained by determining which powers of the basis elements sum to zero.

Table 12. Weight Distribution of an Unusual (2040.32) Binary Code

Spectrum 82-85, Dual Basis (61 122 244 233 211 167 79 158)

wt	count	wt	count	wt	count	wt	count
680	24	966	19524840	1026	328892880	1088	1433865
850	72	968	7900920	1030	309106920	1090	2731560
908	2040	972	29794200	1032	98811480	1094	1587120
914	4080	976	19927485	1036	221435880	1096	371280
918	10200	978	60078000	1040	89637600	1100	491640
920	4080	982	82648560	1042	212631240	1104	112200
924	23460	984	31907640	1046	175962240	1106	248880
928	32640	988	104168520	1048	52421880	1110	116280
930	142800	992	61817610	1052	104401080	1112	16320
934	206040	994	176103000	1056	37531070	1116	25500
936	110160	998	212765880	1058	82530240	1120	10200
940	503880	1000	77089560	1062	60859320	1122	10200
944	448800	1004	221454240	1064	17307360	1126	4080
946	1646280	1008	115000410	1068	29526960	1132	2040
950	2913120	1010	310412520	1072	9266190	1136	2040
952	1287240	1014	329731320	1074	19881840	1190	72
956	5183640	1016	112059240	1078	12523560	1360	27
960	3863335	1020	284335260	1080	3366000		
962	12645960	1024	130678575	1084	5284620		

For example, from the diagrams in [5], any expansion of the (255,4) RS code with spectrum (3-6) must produce codewords whose weights are divisible by 8. However, if the basis satisfies

$$\sum_{i=0}^{m-1} \beta_i^e \;=\; 0 \qquad \text{for} \qquad e = 43, 45, 51, 53, 85 \tag{5}$$

then the weights of all codewords will be divisible by 16. From the table of power sums in [5], the only normal bases that satisfy (5) are the 8-th, 9-th, and 10-th normal bases (those based on α^{43}, α^{47}, and α^{53}). Examination of the calculated weight distributions shows that these three bases resulted in gaps of size 16, while all the other expansions resulted in gaps of size 8.

Almost all of the regular gaps in the weight distributions can be explained in this way. However, there are a few cases that are more complex. For example, expansions of (255,4) RS codes whose spectra contain frequency 15 must have weights divisible by 2. Many of the expansions can be shown to have gaps of size 4 by using the tables in [5]. But a few of the calculated distributions have gaps of size 8.

While most of the gaps in the central part of the distributions are simple powers of 2, a few codes have much more irregular patterns of gaps. The most unusual of these is the expansion of the (255,4) Reed-Solomon code with spectrum (82-85) using the normal basis (61,122,244,233,211,167,79,158). The resulting weight distribution is shown in Table 12.

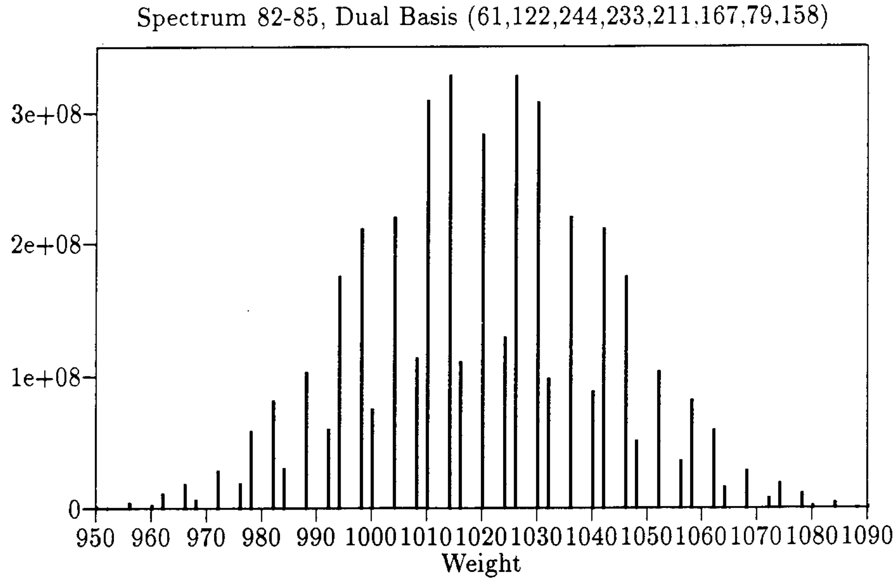

Figure 4. Weight Distribution of an Unusual (2040,32) Binary Code

Even in the central part of the distribution, the size of the gaps varies between 2 and 4. The gaps are symmetric about weight 1020 (which is n/2) and have other symmetries, which can be seen in Figure 4. The number of codewords in the figure is plotted with a linear scale to show that the weight distribution resembles two normal distributions — the lower one consists of all weights that are divisible by 8, and the upper one consists of the other weights. It is extremely unusual for the central part of a weight distribution to look so much different from a normal distribution, although a few other codes in this study also resemble two or more overlapping normal distributions. These cases are now being investigated.

5 Conclusions

This study has examined the weight distributions of 3064 binary codes derived by expanding low rate Reed-Solomon codes with various bases. Almost all the resulting binary codes have minimum distances far greater than the minimum distances of the original Reed-Solomon codes and close to the parameters of BCH codes with similar sizes. All the minimum distances are listed in Tables B-1 through B-7.

The Poltyrev bound on the probability of error using a maximum likelihood decoder was calculated from each of the weight distributions. This showed that most of the codes are capable of decoded error rates very close to those of randomly chosen codes or randomly chosen GRS codes. It also showed that the minimum distance of one of these codes is a good measure of its error-correction capability with a maximum likelihood decoder on a binary symmetric channel, even when the channel is very noisy. The weight distributions computed in this study make it possible to choose the best combination of RS spectrum and basis for use with either maximum likelihood or bounded distance decoding. They may also be useful in choosing codes for use with other types of decoders.

The numerical weight distributions of all 3064 codes are available from the author. Small graphs of the distributions are included in Appendix C. From these graphs, interesting patterns can be observed. The pattern of gaps in the weight distributions was compared with the theorem in [5], which explains most of the gaps. However, a few of the more unusual cases remain to be explained.

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Appendix A Log Tables

Table A-1. Log Table for GF(32)

exp	T	poly	exp	T	poly	exp	T	poly	exp	T	poly
0	1	00001	8	0	01101	16	0	11011	24	1	11110
1	0	00010	9	1	11010	17	1	10011	25	0	11001
2	0	00100	10	1	10001	18	1	00011	26	1	10111
3	1	01000	11	1	00111	19	0	00110	27	0	01011
4	0	10000	12	1	01110	20	1	01100	28	0	10110
5	1	00101	13	1	11100	21	1	11000	29	0	01001
6	1	01010	14	0	11101	22	1	10101	30	0	10010
7	0	10100	15	0	11111	23	0	01111			

Table A-2. Log Table for GF(64)

exp	T	poly	exp	T	poly	exp	T	poly	exp	T	poly
0	0	000001	16	0	010011	32	0	001001	48	0	001101
1	0	000010	17	1	100110	33	0	010010	49	0	011010
2	0	000100	18	0	001111	34	1	100100	50	1	110100
3	0	001000	19	0	011110	35	0	001011	51	1	101011
4	0	010000	20	1	111100	36	0	010110	52	0	010101
5	1	100000	21	1	111011	37	1	101100	53	1	101010
6	0	000011	22	1	110101	38	0	011011	54	0	010111
7	0	000110	23	1	101001	39	1	110110	55	1	101110
8	0	001100	24	0	010001	40	1	101111	56	0	011111
9	0	011000	25	1	100010	41	0	011101	57	1	111110
10	1	110000	26	0	000111	42	1	111010	58	1	111111
11	1	100011	27	0	001110	43	1	110111	59	1	111101
12	0	000101	28	0	011100	44	1	101101	60	1	111001
13	0	001010	29	1	111000	45	0	011001	61	1	110001
14	0	010100	30	1	110011	46	1	110010	62	1	100001
15	1	101000	31	1	100101	47	1	100111			

Table A-3. Log Table for GF(128)

exp	T	poly	exp	T	poly	exp	T	poly	exp	T	poly
0	1	0000001	32	0	0010110	64	0	0010010	96	0	1001010
1	0	0000010	33	0	0101100	65	0	0100100	97	1	0010111
2	0	0000100	34	0	1011000	66	0	1001000	98	0	0101110
3	0	0001000	35	1	0110011	67	1	0010011	99	0	1011100
4	0	0010000	36	0	1100110	68	0	0100110	100	1	0111011
5	0	0100000	37	1	1001111	69	0	1001100	101	0	1110110
6	0	1000000	38	1	0011101	70	1	0011011	102	1	1101111
7	1	0000011	39	0	0111010	71	0	0110110	103	1	1011101
8	0	0000110	40	0	1110100	72	0	1101100	104	1	0111001
9	0	0001100	41	1	1101011	73	1	1011011	105	0	1110010
10	0	0011000	42	1	1010101	74	1	0110101	106	1	1100111
11	0	0110000	43	1	0101001	75	0	1101010	107	1	1001101
12	0	1100000	44	0	1010010	76	1	1010111	108	1	0011001
13	1	1000011	45	1	0100111	77	1	0101101	109	0	0110010
14	1	0000101	46	0	1001110	78	0	1011010	110	0	1100100
15	0	0001010	47	1	0011111	79	1	0110111	111	1	1001011
16	0	0010100	48	0	0111110	80	0	1101110	112	1	0010101
17	0	0101000	49	0	1111100	81	1	1011111	113	0	0101010
18	0	1010000	50	1	1111011	82	1	0111101	114	0	1010100
19	1	0100011	51	1	1110101	83	0	1111010	115	1	0101011
20	0	1000110	52	1	1101001	84	1	1110111	116	0	1010110
21	1	0001111	53	1	1010001	85	1	1101101	117	1	0101111
22	0	0011110	54	1	0100001	86	1	1011001	118	0	1011110
23	0	0111100	55	0	1000010	87	1	0110001	119	1	0111111
24	0	1111000	56	1	0000111	88	0	1100010	120	0	1111110
25	1	1110011	57	0	0001110	89	1	1000111	121	1	1111111
26	1	1100101	58	0	0011100	90	1	0001101	122	1	1111101
27	1	1001001	59	0	0111000	91	0	0011010	123	1	1111001
28	1	0010001	60	0	1110000	92	0	0110100	124	1	1110001
29	0	0100010	61	1	1100011	93	0	1101000	125	1	1100001
30	0	1000100	62	1	1000101	94	1	1010011	126	1	1000001
31	1	0001011	63	1	0001001	95	1	0100101			

Table A-4. Log Table for GF(256)

exp	T	poly	exp	T	poly	exp	T	poly	exp	T	poly	exp	T	poly
0	0	00000001	51	0	00001010	102	0	01000100	153	0	10010010	204	0	11011101
1	0	00000010	52	0	00010100	103	0	10001000	154	1	00111001	205	1	10100111
2	0	00000100	53	1	00101000	104	0	00001101	155	1	01110010	206	0	01010011
3	0	00001000	54	0	01010000	105	0	00011010	156	1	11100100	207	1	10100110
4	0	00010000	55	1	10100000	106	1	00110100	157	0	11010101	208	0	01010001
5	1	00100000	56	0	01011101	107	1	01101000	158	1	10110111	209	1	10100010
6	0	01000000	57	1	10111010	108	0	11010000	159	1	01110011	210	0	01011001
7	0	10000000	58	1	01101001	109	1	10111101	160	1	11100110	211	1	10110010
8	0	00011101	59	0	11010010	110	1	01100111	161	0	11010001	212	1	01111001
9	1	00111010	60	1	10111001	111	0	11001110	162	1	10111111	213	1	11110010
10	1	01110100	61	1	01101111	112	0	10000001	163	1	01100011	214	1	11111001
11	1	11101000	62	0	11011110	113	0	00011111	164	0	11000110	215	1	11101111
12	0	11001101	63	1	10100001	114	1	00111110	165	0	10010001	216	0	11000011
13	0	10000111	64	0	01011111	115	1	01111100	166	1	00111111	217	0	10011011
14	0	00010011	65	1	10111110	116	1	11111000	167	1	01111110	218	1	00101011
15	1	00100110	66	1	01100001	117	1	11101101	168	1	11111100	219	0	01010110
16	0	01001100	67	0	11000010	118	0	11000111	169	1	11100101	220	1	10101100
17	0	10011000	68	0	10011001	119	0	10010011	170	0	11010111	221	0	01000101
18	1	00101101	69	1	00101111	120	1	00111011	171	1	10110011	222	0	10001010
19	0	01011010	70	0	01011110	121	1	01110110	172	1	01111011	223	0	00001001
20	1	10110100	71	1	10111100	122	1	11101100	173	1	11110110	224	0	00010010
21	1	01110101	72	1	01100101	123	0	11000101	174	1	11110001	225	1	00100100
22	1	11101010	73	0	11001010	124	0	10010111	175	1	11111111	226	0	01001000
23	0	11001001	74	0	10001001	125	1	00110011	176	1	11100011	227	0	10010000
24	0	10001111	75	0	00001111	126	1	01100110	177	0	11011011	228	1	00111101
25	0	00000011	76	0	00011110	127	0	11001100	178	1	10101011	229	1	01111010
26	0	00000110	77	1	00111100	128	0	10000101	179	0	01001011	230	1	11110100
27	0	00001100	78	1	01111000	129	0	00010111	180	0	10010110	231	1	11110101
28	0	00011000	79	1	11110000	130	1	00101110	181	1	00110001	232	1	11110111
29	1	00110000	80	1	11111101	131	0	01011100	182	1	01100010	233	1	11110011
30	1	01100000	81	1	11100111	132	1	10111000	183	0	11000100	234	1	11111011
31	0	11000000	82	0	11010011	133	1	01101101	184	0	10010101	235	1	11101011
32	0	10011101	83	1	10111011	134	0	11011010	185	1	00110111	236	0	11001011
33	1	00100111	84	1	01101011	135	1	10101001	186	1	01101110	237	0	10001011
34	0	01001110	85	0	11010110	136	0	01001111	187	0	11011100	238	0	00001011
35	0	10011100	86	1	10110001	137	0	10011110	188	1	10100101	239	0	00010110
36	1	00100101	87	1	01111111	138	1	00100001	189	0	01010111	240	1	00101100
37	0	01001010	88	1	11111110	139	0	01000010	190	1	10101110	241	0	01011000
38	0	10010100	89	1	11100001	140	0	10000100	191	0	01000001	242	1	10110000
39	1	00110101	90	0	11011111	141	0	00010101	192	0	10000010	243	1	01111101
40	1	01101010	91	1	10100011	142	1	00101010	193	0	00011001	244	1	11111010
41	0	11010100	92	0	01011011	143	0	01010100	194	1	00110010	245	1	11101001
42	1	10110101	93	1	10110110	144	1	10101000	195	1	01100100	246	0	11001111
43	1	01110111	94	1	01110001	145	0	01001101	196	0	11001000	247	0	10000011
44	1	11101110	95	1	11100010	146	0	10011010	197	0	10001101	248	0	00011011
45	0	11000001	96	0	11011001	147	1	00101001	198	0	00000111	249	1	00110110
46	0	10011111	97	1	10101111	148	0	01010010	199	0	00001110	250	1	01101100
47	1	00100011	98	0	01000011	149	1	10100100	200	0	00011100	251	0	11011000
48	0	01000110	99	0	10000110	150	0	01010101	201	1	00111000	252	1	10101101
49	0	10001100	100	0	00010001	151	1	10101010	202	1	01110000	253	0	01000111
50	0	00000101	101	1	00100010	152	0	01001001	203	1	11100000	254	0	10001110

Appendix B Minimum Distance Tables

Table B-1. Minimum Distances of (155,35) Codes

	Dual Basis				
	p	n1	n2	n3	STK
	0	3	5	11	b
	1	6	10	22	o
	2	12	20	13	u
	3	24	9	26	n
Spectrum	4	17	18	21	d
1-7	44	44	40	40	30
2-8	42	44	40	40	30
3-9	40	40	40	40	30
4-10	42	40	40	40	30
5-11	40	40	40	44	20
6-12	42	40	40	44	20
7-13	42	42	40	42	20
8-14	42	40	40	40	20
9-15	42	42	40	40	20
10-16	42	40	40	40	10
11-17	42	42	40	40	20
12-18	42	42	44	44	10

Table B-2. Minimum Distances of (378,36) Codes

	Dual Basis					
	p	n1	n2	n3	n4	
	0	5	15	23	31	STK
	1	10	30	46	62	b
	2	20	60	29	61	o
	3	40	57	58	59	u
	4	17	51	53	55	n
Spectrum	5	34	39	43	47	d
1-6	128	128	128	128	128	96
2-7	134	132	128	132	132	84
3-8	134	132	128	136	132	84
4-9	132	132	108	132	132	84
5-10	132	128	108	136	132	84
6-11	128	136	108	132	136	72
7-12	130	136	108	132	132	72
8-13	130	134	108	136	126	72
9-14	128	130	108	136	132	72
10-15	130	132	120	136	132	60
11-16	130	132	120	132	134	48
12-17	132	132	120	136	132	48
13-18	128	132	108	132	128	60
14-19	132	136	108	128	136	60
15-20	132	128	108	132	136	72
16-21	126	126	84	132	126	84
17-22	126	126	84	132	126	84
18-23	126	126	84	120	120	60
19-24	126	126	84	120	126	60
20-25	126	126	84	120	126	72
21-26	126	126	84	136	120	72
22-27	128	108	108	132	134	72
23-28	130	108	108	132	120	60
24-29	132	108	108	128	134	60
25-30	130	132	108	128	136	72
26-31	132	132	108	132	136	72
27-32	132	128	108	132	134	72
28-33	130	108	126	132	120	48
29-34	128	108	126	130	120	48

Table B-3. Minimum Distances of (889,35) Codes

	Dual Basis								
	p	n1	n2	n3	n4	n5	n6	n7	
	0	13	19	21	27	31	43	63	STK
	1	26	38	42	54	62	86	126	
	2	52	76	84	108	124	45	125	b
	3	104	25	41	89	121	90	123	o
	4	81	50	82	51	115	53	119	u
	5	35	100	37	102	103	106	111	n
Spectrum	6	70	73	74	77	79	85	95	d
1-5	320	320	320	320	320	320	320	320	320
2-6	344	352	336	336	352	352	344	344	336
3-7	356	360	364	360	352	364	364	356	308
4-8	360	368	368	360	360	364	364	360	308
5-9	360	364	360	360	360	364	360	364	224
6-10	364	360	360	368	360	364	364	364	224
7-11	360	368	360	364	360	336	364	360	224
8-12	360	364	352	364	364	364	360	360	224
9-13	360	360	364	356	360	360	364	364	252
10-14	360	360	336	364	364	364	360	364	252
11-15	354	350	364	362	364	364	368	364	238
12-16	358	350	364	368	360	360	364	364	224
13-17	360	364	336	354	368	350	356	364	238
14-18	358	362	368	364	368	350	364	368	252
15-19	356	364	360	368	356	360	364	364	252
16-20	360	356	360	368	364	364	360	364	256
17-21	360	364	360	364	364	360	364	360	252
18-22	360	364	364	356	364	364	356	360	252
19-23	356	360	368	360	368	364	364	360	224
20-24	360	364	360	364	364	364	360	364	224
21-25	356	364	364	364	360	360	364	364	252
22-26	360	360	364	364	360	356	356	362	238
23-27	356	364	356	364	364	360	360	362	224
24-28	358	364	360	360	360	364	364	366	238
25-29	360	368	356	364	360	360	368	364	210
26-30	356	358	360	364	336	362	364	336	224
27-31	360	364	360	364	360	366	364	364	224
28-32	360	360	360	364	364	362	364	360	196
29-33	352	360	364	364	364	364	364	368	196
30-34	360	356	356	368	364	364	360	364	224
31-35	360	360	360	352	360	360	360	364	224

Table B-4. Minimum Distances of (889,35) Codes

	Dual Basis								
	p	n1	n2	n3	n4	n5	n6	n7	
	0	13	19	21	27	31	43	63	STK
	1	26	38	42	54	62	86	126	
	2	52	76	84	108	124	45	125	b
	3	104	25	41	89	121	90	123	o
	4	81	50	82	51	115	53	119	u
	5	35	100	37	102	103	106	111	n
Spectrum	6	70	73	74	77	79	85	95	d
32-36	360	336	364	364	360	368	368	364	252
33-37	360	336	368	364	368	364	368	364	256
34-38	360	364	364	360	364	360	364	364	252
35-39	360	360	364	364	348	360	356	360	196
36-40	356	364	364	360	364	364	364	364	224
37-41	360	360	356	356	360	360	360	360	252
38-42	360	364	368	360	356	360	364	356	252
39-43	350	356	360	364	366	360	364	362	280
40-44	360	356	364	364	362	348	364	366	266
41-45	344	338	348	340	346	340	344	348	270
42-46	352	366	360	336	366	368	364	354	280
43-47	360	368	364	336	364	356	360	356	280
44-48	360	368	364	336	364	364	368	360	224
45-49	360	364	368	336	360	364	360	356	224
46-50	356	364	360	364	360	360	356	364	196
47-51	354	364	364	364	360	364	364	356	196
48-52	352	360	350	364	360	364	364	362	238
49-53	356	356	362	360	360	358	368	360	224
50-54	360	356	356	360	360	364	360	362	252
51-55	352	364	368	368	356	364	364	368	252
52-56	356	360	368	368	364	336	364	364	196
53-57	356	364	336	364	364	364	364	364	224
54-58	360	364	336	360	364	360	364	360	252
55-59	360	360	336	360	336	360	364	360	256
56-60	360	362	352	364	336	336	356	368	224
57-61	360	360	360	360	360	336	364	360	224
58-62	356	364	364	364	364	336	360	360	252
59-63	360	364	360	352	368	360	364	364	224
60-64	358	364	350	362	350	366	356	362	196
61-65	358	356	360	360	364	350	364	356	238

Table B-5. Minimum Distances of (2040,32) Codes

	Dual Basis																	
	p	n1	n2	n3	n4	n5	n6	n7	n8	n9	n10	n11	n12	n13	n14	n15	n16	rp
S	0	5	9	11	15	21	29	39	43	47	53	55	61	63	87	91	95	0
p	1	10	18	22	30	42	58	78	86	94	106	110	122	126	174	182	190	85
e	2	20	36	44	60	84	116	156	172	188	212	220	244	252	93	109	125	51
c	3	40	72	88	120	168	232	57	89	121	169	185	233	249	186	218	250	136
t	4	80	144	176	240	81	209	114	178	242	83	115	211	243	117	181	245	15
r	5	160	33	97	225	162	163	228	101	229	166	230	167	231	234	107	235	100
u	6	65	66	194	195	69	71	201	202	203	77	205	79	207	213	214	215	66
m	7	130	132	133	135	138	142	147	149	151	154	155	158	159	171	173	175	151
1-4	768	768	768	768	768	768	768	768	768	768	768	768	768	768	768	768	768	768
2-5	896	896	896	896	896	896	896	896	896	896	896	896	896	896	896	896	896	896
3-6	840	888	864	888	864	864	888	888	864	864	864	864	888	888	864	864	888	864
4-7	904	896	900	912	904	896	896	896	912	912	896	904	896	912	896	912	912	900
5-8	896	896	892	912	904	896	896	908	896	896	912	896	896	904	896	912	904	900
6-9	888	900	896	896	864	904	896	864	896	912	896	912	904	896	864	904	888	896
7-10	904	896	912	896	904	896	896	896	896	912	896	912	904	896	912	896	880	888
8-11	896	896	912	896	904	896	896	912	912	912	912	896	896	896	896	912	896	904
9-12	904	896	864	896	864	864	896	896	896	864	864	864	896	896	904	904	904	888
10-13	904	904	888	908	912	864	896	904	896	912	912	896	896	912	912	900	912	896
11-14	896	908	920	896	880	864	908	908	904	896	896	900	896	904	896	904	896	892
12-15	900	840	816	720	720	840	720	840	720	864	896	840	896	840	840	908	840	840
13-16	902	840	840	720	720	840	720	840	720	896	896	840	896	840	840	912	840	840
14-17	902	840	840	720	720	840	720	840	720	896	904	840	912	840	840	896	840	544
15-18	894	840	816	720	720	816	720	840	720	896	864	840	896	840	840	768	840	544
16-19	904	896	912	912	880	896	896	896	912	904	912	896	912	896	896	896	904	544
17-20	888	896	916	896	864	896	908	880	912	904	896	908	896	896	896	904	904	544
18-21	908	864	892	896	864	896	888	864	888	896	864	864	904	896	896	912	896	852
19-22	900	896	896	896	904	896	896	912	896	896	912	888	904	896	888	912	908	892
20-23	900	896	864	896	896	896	896	896	832	904	896	896	896	896	888	912	896	872
21-24	888	864	896	896	904	864	904	904	864	912	904	864	904	912	864	864	832	888
22-25	904	908	896	888	912	912	912	904	908	896	904	896	896	888	864	908	904	884
23-26	900	896	896	908	916	896	896	912	900	912	904	908	864	896	900	912	912	872
24-27	896	864	896	864	832	888	896	900	904	768	908	908	896	768	904	908	896	864
25-28	904	896	908	904	864	908	896	904	904	912	912	908	904	864	912	912	908	904
26-29	896	848	892	896	880	904	896	864	884	888	904	896	912	896	904	912	908	900
27-30	896	840	840	720	720	816	720	840	720	888	864	840	864	840	840	896	840	840
28-31	896	840	840	720	720	840	720	840	720	906	896	840	904	840	840	896	840	840
29-32	904	840	840	720	720	840	720	840	720	904	912	840	904	840	840	904	840	840
30-33	900	840	816	720	720	816	720	840	720	864	888	840	904	840	840	904	840	840
31-34	896	896	880	912	888	888	904	896	904	904	896	896	904	896	896	896	896	544
32-35	904	912	912	912	896	904	896	896	912	912	896	896	912	832	904	904	904	544
33-36	840	864	864	864	856	864	864	864	848	856	848	864	864	864	864	864	864	544
34-37	896	896	900	908	904	908	912	896	880	912	896	908	908	896	880	904	896	544
35-38	896	832	904	912	912	912	904	880	892	896	900	912	904	896	912	908	896	876
36-39	908	896	864	864	896	904	896	896	880	896	900	908	904	896	912	864	864	888
37-40	892	896	896	896	900	908	896	896	904	908	896	904	912	896	896	904	864	896
38-41	904	904	904	896	896	864	908	904	896	896	896	896	904	896	896	904	904	900
39-42	900	864	896	896	864	908	896	864	912	896	912	904	888	896	896	896	896	900
40-43	900	896	896	912	864	864	896	892	912	912	896	912	896	896	896	896	896	896
41-44	896	896	908	892	880	864	896	880	896	880	880	896	896	864	892	896	884	876
42-45	906	840	840	720	720	840	720	840	720	896	832	840	864	816	840	912	840	840

Table B-6. Minimum Distances of (2040,32) Codes

	Dual Basis																	
	p	n1	n2	n3	n4	n5	n6	n7	n8	n9	n10	n11	n12	n13	n14	n15	n16	rp
S	0	5	9	11	15	21	29	39	43	47	53	55	61	63	87	91	95	0
p	1	10	18	22	30	42	58	78	86	94	106	110	122	126	174	182	190	85
e	2	20	36	44	60	84	116	156	172	188	212	220	244	252	93	109	125	51
c	3	40	72	88	120	168	232	57	89	121	169	185	233	249	186	218	250	136
t	4	80	144	176	240	81	209	114	178	242	83	115	211	243	117	181	245	15
r	5	160	33	97	225	162	163	228	101	229	166	230	167	231	234	107	235	100
u	6	65	66	194	195	69	71	201	202	203	77	205	79	207	213	214	215	66
m	7	130	132	133	135	138	142	147	149	151	154	155	158	159	171	173	175	151
43-46	896	840	840	720	720	840	720	840	720	896	896	840	896	840	840	896	840	840
44-47	906	840	840	720	720	840	720	840	720	896	912	840	896	840	840	896	840	840
45-48	810	840	840	720	720	840	720	840	720	912	896	840	888	840	840	904	840	840
46-49	900	908	896	896	880	896	912	900	904	912	904	896	896	832	864	896	912	884
47-50	900	904	904	912	832	904	908	908	912	908	896	896	904	832	908	904	904	848
48-51	714	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	408
49-52	714	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	408
50-53	714	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	408
51-54	714	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	408
52-55	900	912	864	908	912	880	904	896	896	896	912	904	912	896	904	896	908	848
53-56	908	908	912	912	880	912	880	896	896	896	896	896	904	896	916	904	912	888
54-57	898	864	906	912	872	896	904	908	904	888	908	912	880	816	906	912	864	900
55-58	900	896	904	904	896	896	908	896	908	904	896	908	912	864	892	912	912	892
56-59	904	864	896	904	880	908	864	832	900	904	912	912	900	864	896	912	900	900
57-60	896	840	840	720	720	840	720	840	720	896	896	840	864	840	816	896	840	840
58-61	896	840	840	720	720	840	720	840	720	912	904	840	912	840	840	896	840	840
59-62	906	840	840	720	720	840	720	840	720	912	908	840	912	840	840	900	840	840
60-63	894	840	816	720	720	840	720	840	720	864	892	840	888	840	840	912	840	840
61-64	904	896	912	912	864	896	832	896	916	908	904	912	904	896	908	864	896	904
62-65	904	896	904	908	896	896	904	896	880	896	912	896	896	896	864	904	912	876
63-66	896	864	896	816	880	864	908	912	912	864	864	864	896	864	896	816	864	896
64-67	896	880	896	912	896	896	896	896	912	896	896	912	896	896	896	912	896	880
65-68	900	908	896	896	916	896	904	896	880	896	904	912	896	912	896	912	896	544
66-69	888	864	904	896	896	908	864	864	912	896	908	864	904	896	904	896	896	544
67-70	908	916	900	912	904	880	904	912	904	896	896	896	880	896	916	896	904	544
68-71	900	920	900	908	912	912	900	916	900	896	912	896	864	908	896	896	896	544
69-72	876	864	768	896	892	904	768	912	896	864	896	864	864	896	896	896	896	852
70-73	904	896	892	904	896	904	896	908	896	880	900	912	912	896	912	908	904	892
71-74	896	832	892	896	880	896	896	896	896	880	896	896	896	832	896	896	896	884
72-75	852	840	840	720	720	840	720	840	720	864	896	840	896	840	840	896	840	840
73-76	896	840	840	720	720	840	720	840	720	888	896	840	896	840	840	896	840	840
74-77	902	840	840	720	720	840	720	840	720	832	908	840	832	840	840	896	840	840
75-78	906	840	840	720	720	840	720	816	720	832	864	840	896	840	840	904	840	840
76-79	880	900	896	896	864	908	888	896	896	832	896	896	896	896	912	880	904	896
77-80	900	908	896	896	896	908	908	908	888	912	896	896	896	896	904	908	904	900
78-81	904	912	896	896	864	900	896	864	896	908	864	896	904	896	832	912	864	896
79-82	904	896	896	896	904	896	896	912	896	912	912	832	912	880	832	912	896	888
80-83	900	908	896	912	896	896	896	912	912	900	864	912	912	880	896	880	896	880
81-84	840	888	888	888	864	888	888	864	840	864	864	864	864	864	888	864	864	744
82-85	510	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	340
83-86	510	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	340
84-87	510	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	340

Table B-7. Minimum Distances of (2040.32) Codes

	Dual Basis																	
	p	n1	n2	n3	n4	n5	n6	n7	n8	n9	n10	n11	n12	n13	n14	n15	n16	rp
S	0	5	9	11	15	21	29	39	43	47	53	55	61	63	87	91	95	0
p	1	10	18	22	30	42	58	78	86	94	106	110	122	126	174	182	190	85
e	2	20	36	44	60	84	116	156	172	188	212	220	244	252	93	109	125	51
c	3	40	72	88	120	168	232	57	89	121	169	185	233	249	186	218	250	136
t	4	80	144	176	240	81	209	114	178	242	83	115	211	243	117	181	245	15
r	5	160	33	97	225	162	163	228	101	229	166	230	167	231	234	107	235	100
u	6	65	66	194	195	69	71	201	202	203	77	205	79	207	213	214	215	66
m	7	130	132	133	135	138	142	147	149	151	154	155	158	159	171	173	175	151
85-88	510	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	680	340
86-89	864	896	888	896	896	896	896	896	896	896	896	896	896	896	896	888	896	768
87-90	910	840	840	720	720	840	720	840	720	908	896	840	896	840	840	904	840	840
88-91	904	840	840	720	720	840	720	840	720	912	896	840	896	840	840	904	840	840
89-92	904	840	840	720	720	840	720	840	720	896	912	840	896	840	840	912	840	840
90-93	876	840	840	720	720	840	720	840	720	864	832	840	896	816	840	896	840	840
91-94	900	896	896	904	872	896	896	832	880	896	904	908	896	896	896	896	896	896
92-95	896	896	896	904	904	896	896	908	896	904	912	912	896	896	896	912	904	892
93-96	896	768	896	904	904	896	896	864	912	904	896	904	864	864	912	896	864	864
94-97	904	912	912	912	896	896	896	896	888	896	896	896	912	896	912	904	896	888
95-98	900	896	896	916	908	864	904	912	904	904	896	896	912	912	912	896	896	904
96-99	896	904	896	864	896	864	912	908	908	904	896	864	896	768	904	904	864	888
97-100	904	908	896	904	896	904	908	896	904	896	904	900	888	896	908	892	896	888
98-101	904	904	912	912	896	908	916	908	896	904	896	896	900	896	904	912	880	856
99-102	714	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	408
100-103	714	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	408
101-104	714	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	816	408
102-105	714	768	816	720	720	816	720	816	720	816	816	816	816	816	816	816	816	408
103-106	900	840	840	720	720	840	720	840	720	908	896	840	904	840	840	896	840	840
104-107	898	840	840	720	720	840	720	840	720	912	904	840	912	840	840	832	840	840
105-108	888	840	840	720	720	840	720	840	720	768	896	840	864	816	840	832	840	840
106-109	896	896	896	896	872	896	896	896	896	896	896	896	896	896	896	896	896	896
107-110	896	896	896	896	896	896	896	896	896	896	896	896	896	896	896	896	896	888
108-111	896	904	864	864	896	904	864	904	896	896	896	864	904	896	912	896	896	888
109-112	900	900	912	908	904	904	908	908	896	904	896	908	896	912	900	908	896	880
110-113	896	896	904	912	896	904	908	912	904	896	896	904	896	896	908	896	896	896
111-114	904	896	896	912	904	864	768	832	904	864	864	896	864	864	916	896	896	904
112-115	908	904	896	920	880	896	896	896	908	912	896	896	904	908	912	904	904	896
113-116	900	904	894	912	904	904	900	900	912	908	908	896	908	896	906	904	896	900
114-117	888	904	896	888	864	864	896	896	916	896	864	864	864	864	912	912	896	864
115-118	900	912	896	896	904	912	896	896	896	896	904	904	912	896	832	912	908	884
116-119	904	908	900	912	872	896	904	908	896	896	908	912	904	896	832	896	904	544
117-120	904	840	840	720	720	816	720	840	720	908	908	840	896	840	840	896	840	544
118-121	906	840	840	720	720	840	720	840	720	912	896	840	896	840	840	904	840	544
119-122	902	840	840	720	720	840	720	840	720	896	904	840	904	840	840	908	840	544
120-123	902	840	840	720	720	840	720	840	720	896	908	840	904	840	816	896	840	840
121-124	900	904	912	896	872	896	832	896	888	896	904	904	904	904	912	896	896	888
122-125	888	904	916	912	896	896	832	896	912	896	900	916	896	904	896	912	912	892
123-126	900	908	896	896	864	912	912	864	864	896	908	768	896	896	864	896	896	888
124-127	904	896	908	896	896	912	892	896	904	896	900	888	896	912	896	896	896	900
125-128	896	908	908	896	910	902	912	896	912	896	908	906	896	896	896	896	896	892
126-129	882	816	902	914	882	906	904	816	816	904	864	912	902	908	880	900	888	864

Table B-8. STK Bound Versus Worst (2040,32) Codes Found

Spectrum	Worst d_{min}	STK Bound
1- 4	768	768
2- 5	896	768
3- 6	840	768
4- 7	896	768
5- 8	892	768
6- 9	864	512
7–10	880	704
8–11	896	640
9–12	864	640
10–13	864	640
11–14	864	640
12–15	720	640
13–16	720	672
14–17	544	544
15–18	544	544
16–19	544	544
17–20	544	544
18–21	852	640
19–22	888	640
20–23	832	640
21–24	832	576
22–25	864	640
23–26	864	640
24–27	768	640
25–28	864	640
26–29	848	640
27–30	720	640
28–31	720	640
29–32	720	640
30–33	720	576
31–34	544	544
32–35	544	544
33–36	544	544
34–37	544	544
35–38	876	704
36–39	864	576
37–40	864	704
38–41	864	704
39–42	864	576
40–43	864	640
41–44	864	640
42–45	720	640

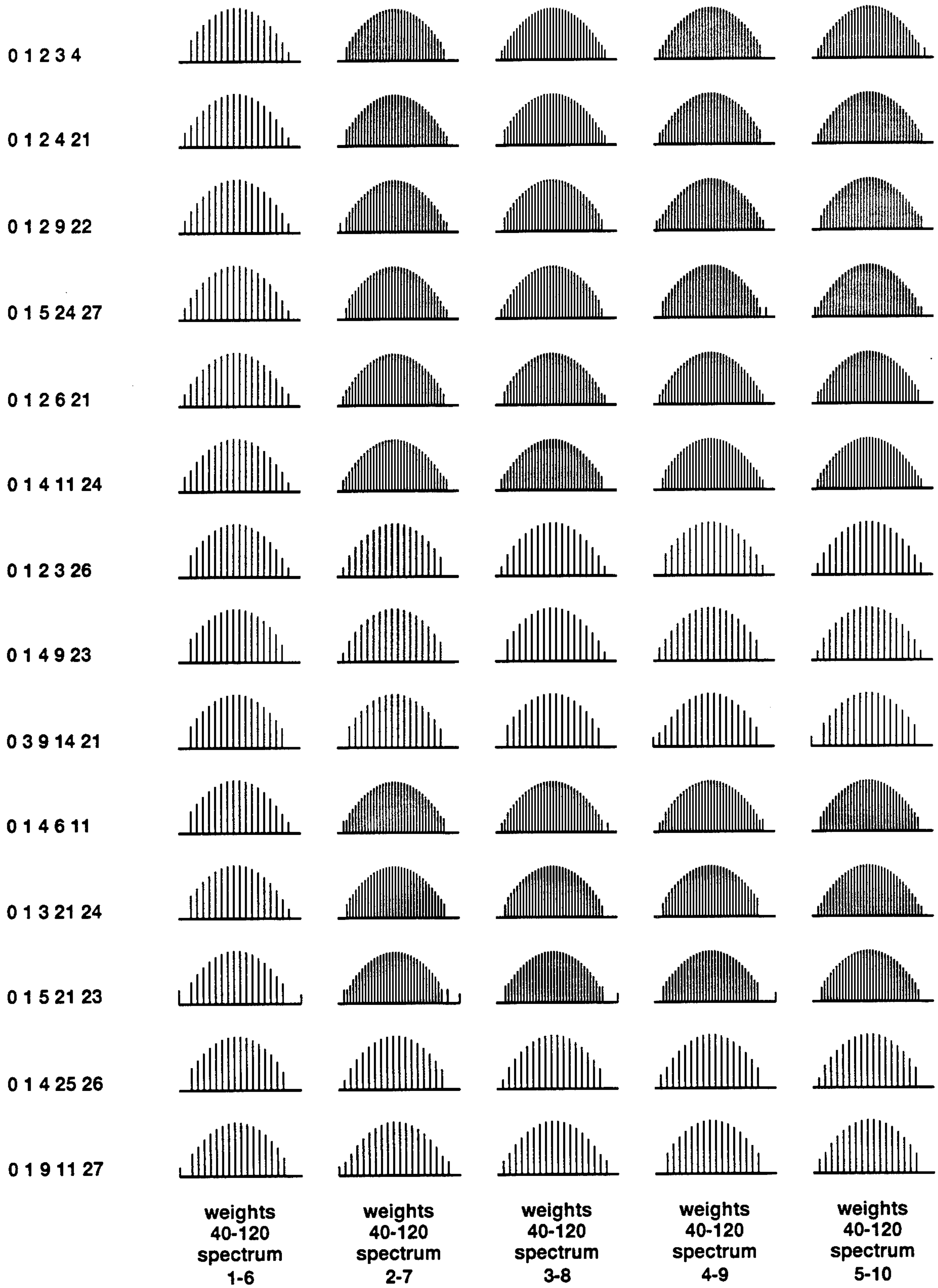
Spectrum	Worst d_{min}	STK Bound
43–46	720	688
44–47	720	640
45–48	720	576
46–49	832	640
47–50	832	576
48–51	408	408
49–52	408	408
50–53	408	408
51–54	408	408
52–55	848	576
53–56	880	640
54–57	864	640
55–58	864	640
56–59	832	640
57–60	720	640
58–61	720	672
59–62	720	704
60–63	720	640
61–64	832	640
62–65	864	704
63–66	816	576
64–67	880	640
65–68	544	544
66–69	544	544
67–70	544	544
68–71	544	544
69–72	768	640
70–73	880	640
71–74	832	640
72–75	720	640
73–76	720	640
74–77	720	672
75–78	720	576
76–79	832	512
77–80	888	640
78–81	832	576
79–82	832	640
80–83	864	640
81–84	744	640
82–85	340	340
83–86	340	340
84–87	340	340

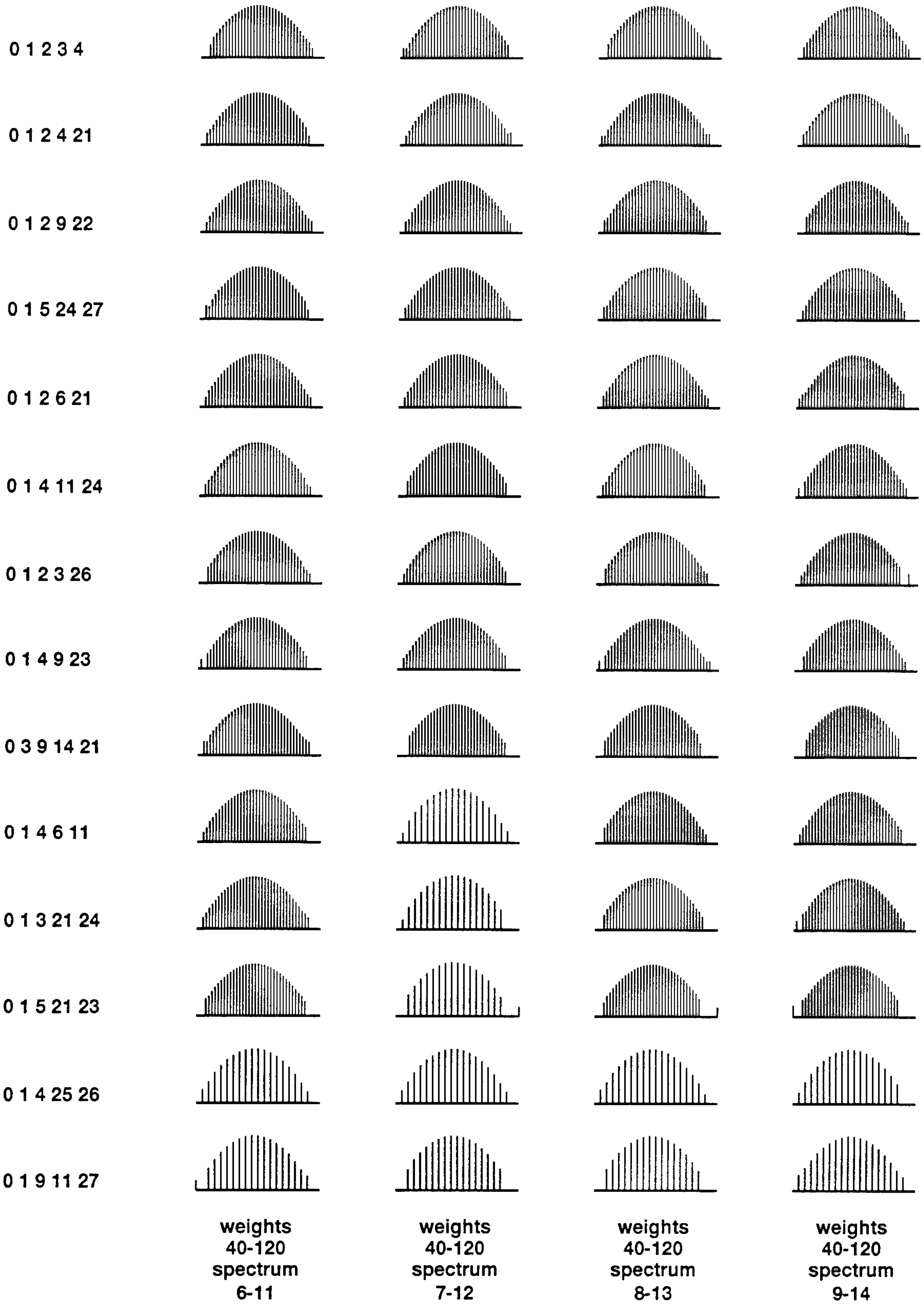
Spectrum	worst d_{min}	SKT Bound
85–88	340	340
86–89	768	640
87–90	720	640
88–91	720	640
89–92	720	704
90–93	720	672
91–94	872	512
92–95	892	640
93–96	768	640
94–97	888	640
95–98	896	640
96–99	768	640
97–100	888	640
98–101	856	576
99–102	408	408
100–103	408	408
101–104	408	408
102–105	408	408
103–106	720	576
104–107	720	688
105–108	720	576
106–109	872	640
107–110	888	640
108–111	864	576
109–112	880	640
110–113	896	640
111–114	832	640
112–115	880	640
113–116	894	640
114–117	864	640
115–118	832	704
116–119	544	544
117–120	544	544
118–121	544	544
119–122	544	544
120–123	720	576
121–124	832	640
122–125	832	672
123–126	768	704
124–127	888	768
125–128	892	736
126–129	816	672

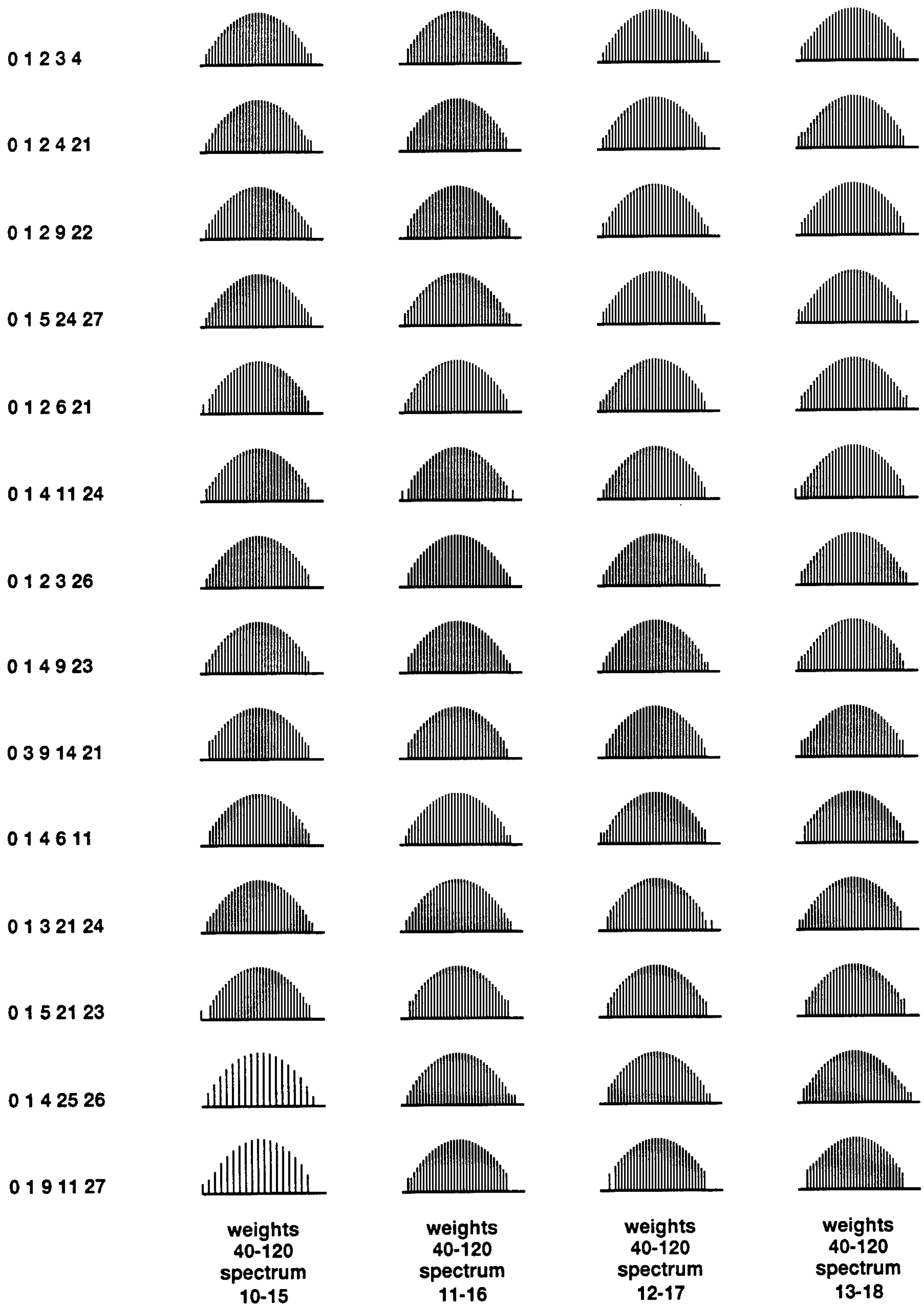
Appendix C Graphs of Weight Distributions

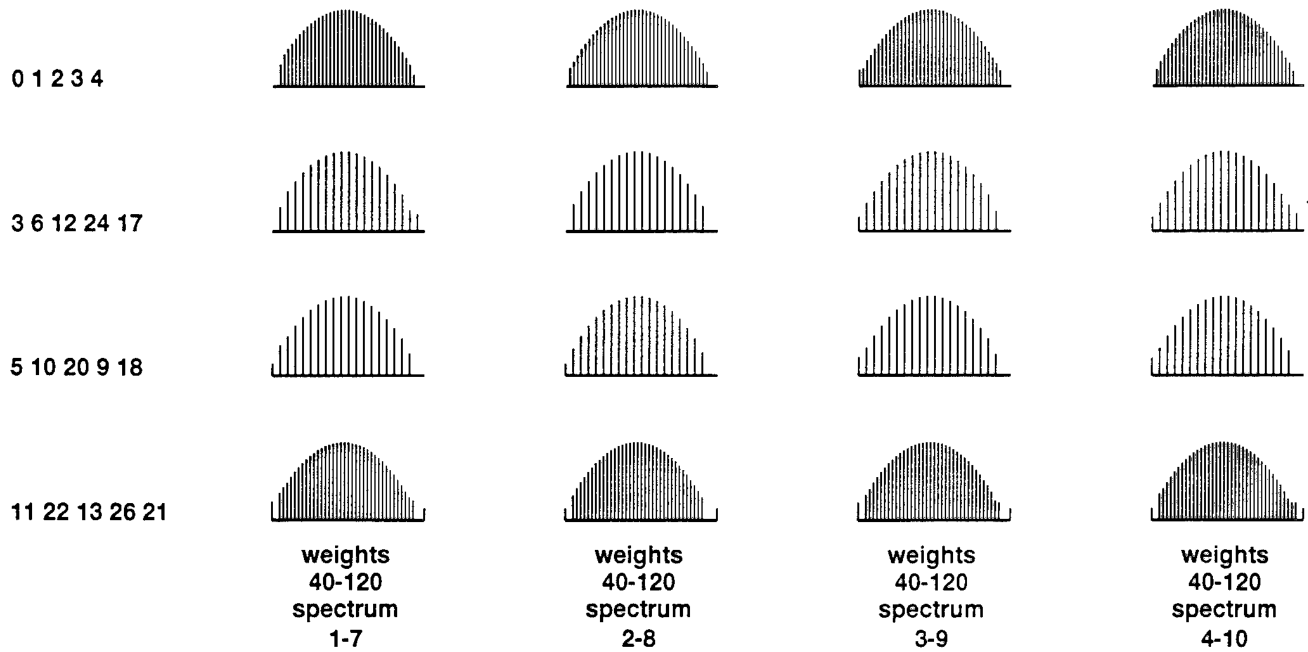
Each of the graphs on the following pages represents the weight distribution of a binary mapping of a Reed-Solomon code, that is, the number of codewords of each possible weight. To save space, the range of the horizontal axis is limited to nonzero weights for which codewords exist, and the range is specified under each column of graphs. The vertical bars have a width of one unit, so a code that contains codewords of each possible weight will produce a solid black graph. The vertical axis shows the log of the number of codewords at each weight.

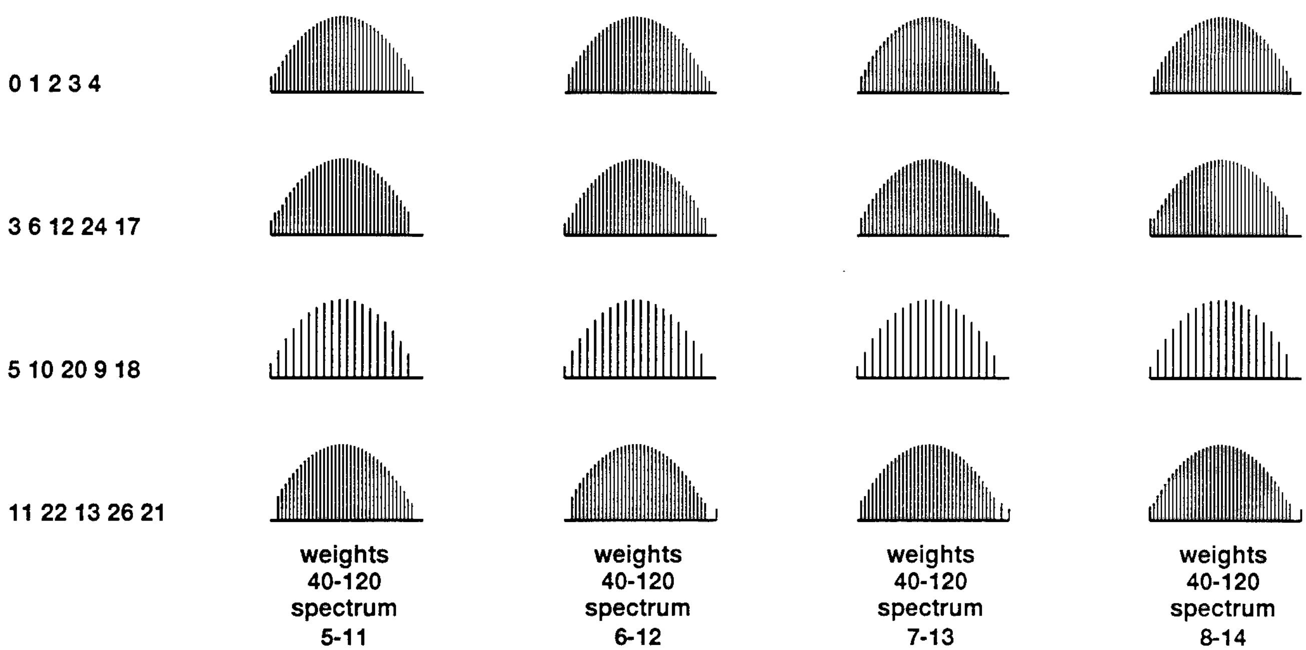
A binary mapping of a Reed-Solomon code is specified by the spectrum of the Reed-Solomon code and the basis used to map the symbols into binary m-tuples. The basis appears at the left of each row, as powers of a primitive element. The spectrum appears at the bottom of each column. Note that the parameters of any of these codes can easily be determined from the basis and spectrum. If m is the number of elements in the basis, the binary block length is $m(2^m - 1)$, and if K is the number of frequencies in the spectrum, the dimension of the binary code is mK.


0 1 2 3 4
0 1 2 4 21
0 1 2 9 22
0 1 5 24 27
0 1 2 6 21
0 1 4 11 24
0 1 2 3 26
0 1 4 9 23
0 3 9 14 21
0 1 4 6 11
0 1 3 21 24
0 1 5 21 23
0 1 4 25 26
0 1 9 11 27
weights 40-120 spectrum 1-6
weights 40-120 spectrum 2-7
weights 40-120 spectrum 3-8
weights 40-120 spectrum 4-9
weights 40-120 spectrum 5-10


0 1 2 3 4
0 1 2 4 21
0 1 2 9 22
0 1 5 24 27
0 1 2 6 21
0 1 4 11 24
0 1 2 3 26
0 1 4 9 23
0 3 9 14 21
0 1 4 6 11
0 1 3 21 24
0 1 5 21 23
0 1 4 25 26
0 1 9 11 27
weights 40-120 spectrum 6-11
weights 40-120 spectrum 7-12
weights 40-120 spectrum 8-13
weights 40-120 spectrum 9-14


0 1 2 3 4
0 1 2 4 21
0 1 2 9 22
0 1 5 24 27
0 1 2 6 21
0 1 4 11 24
0 1 2 3 26
0 1 4 9 23
0 3 9 14 21
0 1 4 6 11
0 1 3 21 24
0 1 5 21 23
0 1 4 25 26
0 1 9 11 27
weights 40-120 spectrum 10-15
weights 40-120 spectrum 11-16
weights 40-120 spectrum 12-17
weights 40-120 spectrum 13-18


0 1 2 3 4
3 6 12 24 17
5 10 20 9 18
11 22 13 26 21
weights 40-120 spectrum 1-7
weights 40-120 spectrum 2-8
weights 40-120 spectrum 3-9
weights 40-120 spectrum 4-10


0 1 2 3 4
3 6 12 24 17
5 10 20 9 18
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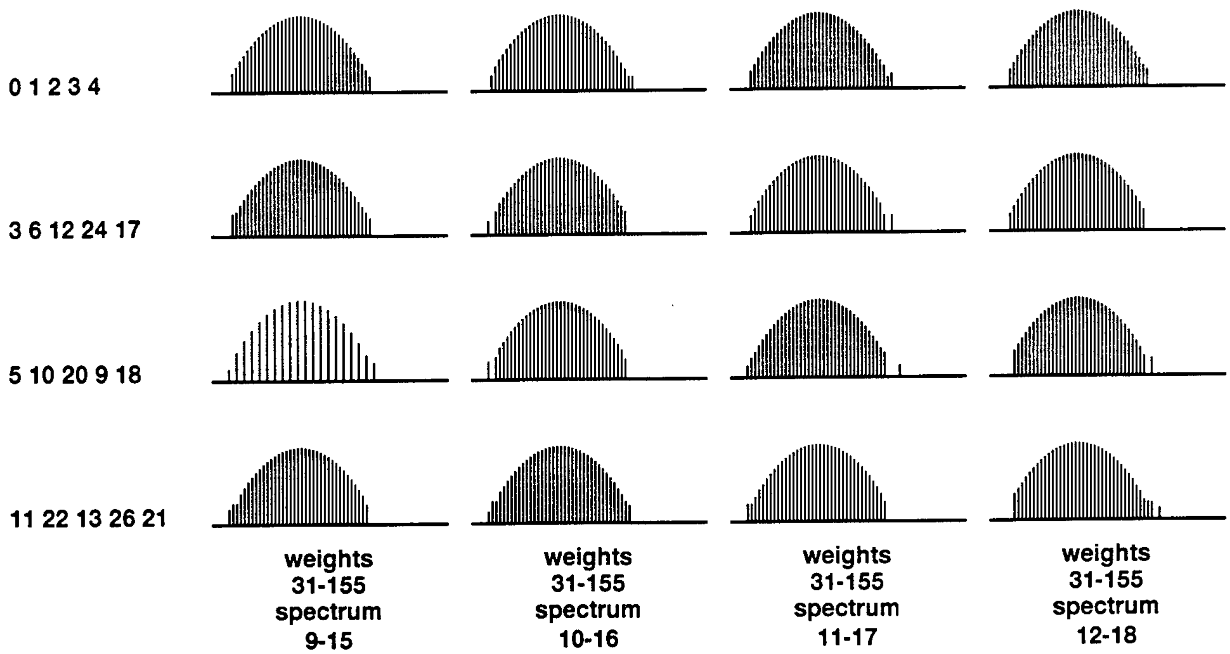
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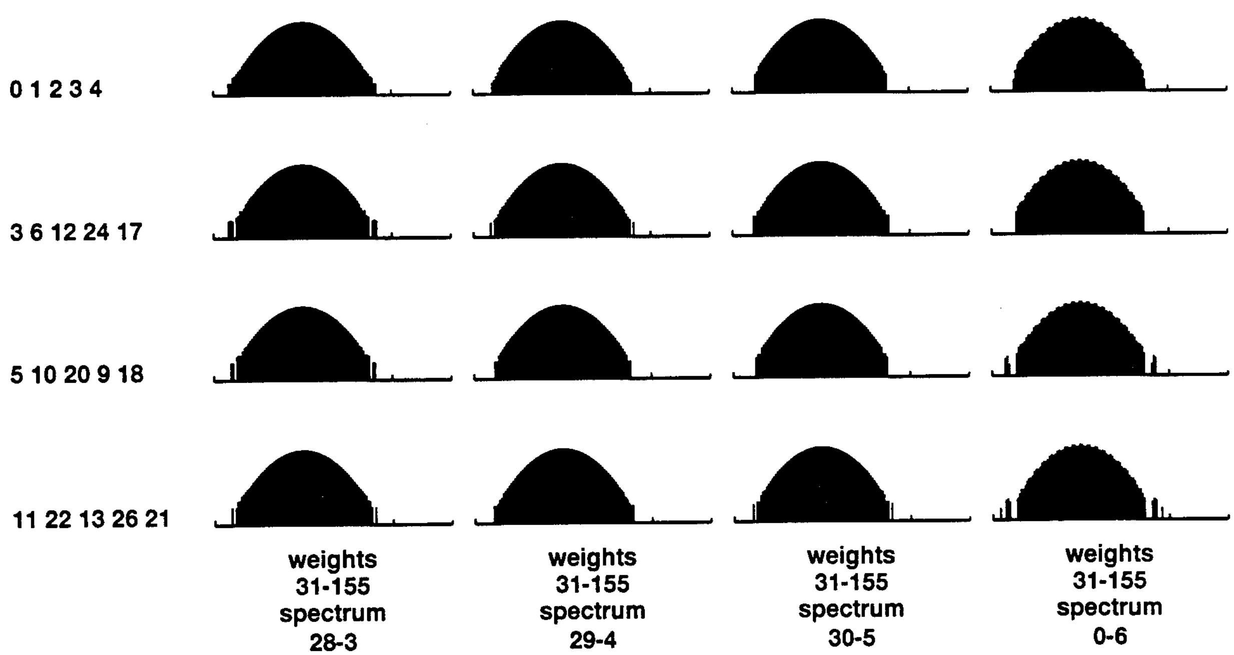
weights 31-155 spectrum 12-18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weights 31-155 spectrum 30-5
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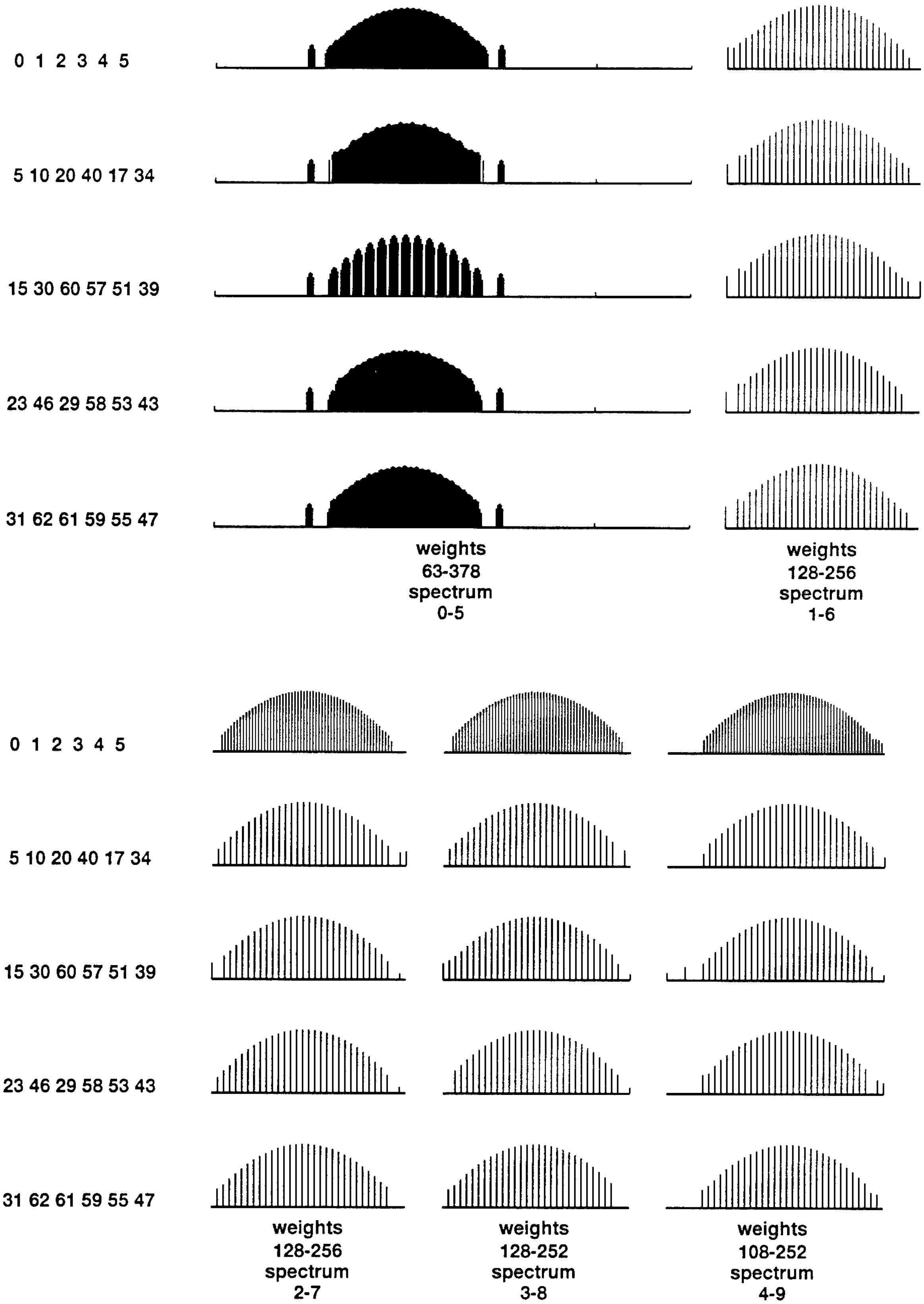
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weights 128-256 spectrum 1-6
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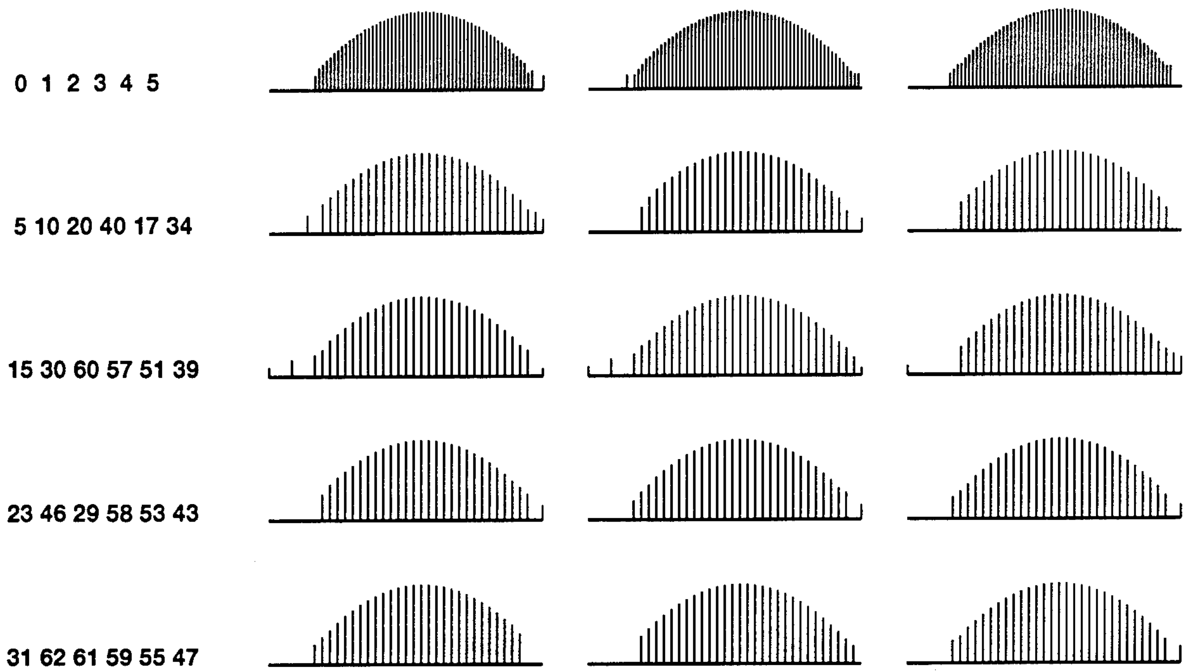
weights 128-252 spectrum 3-8
weights 108-252 spectrum 4-9



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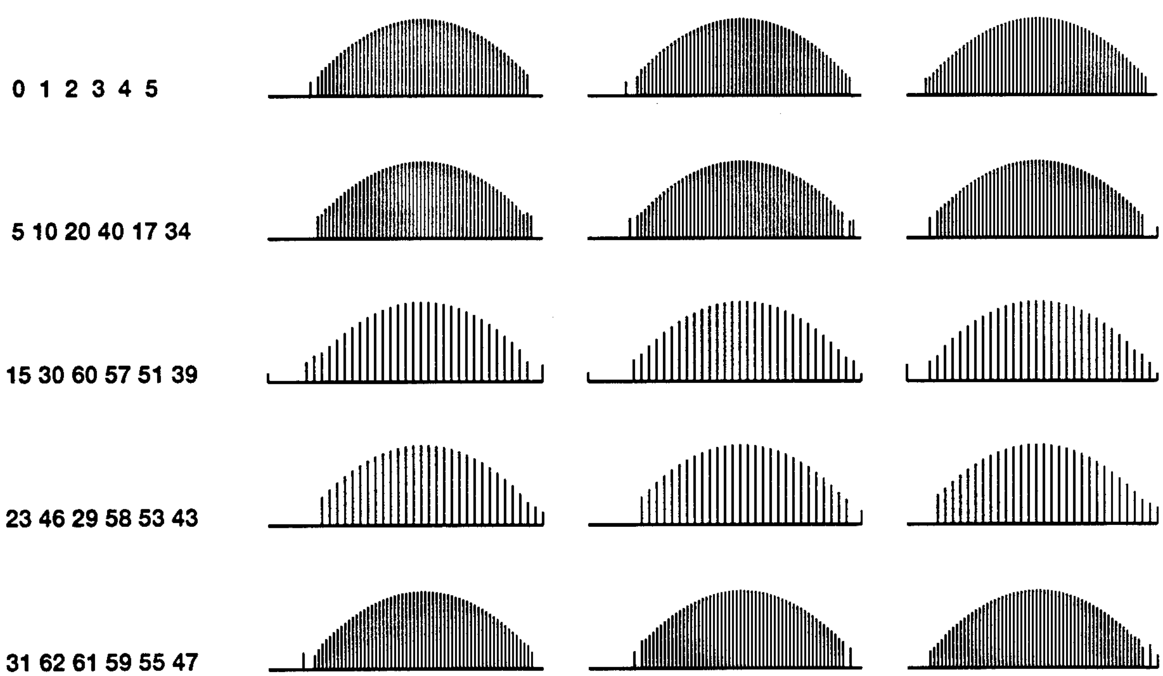

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weights 108-252 spectrum 9-14

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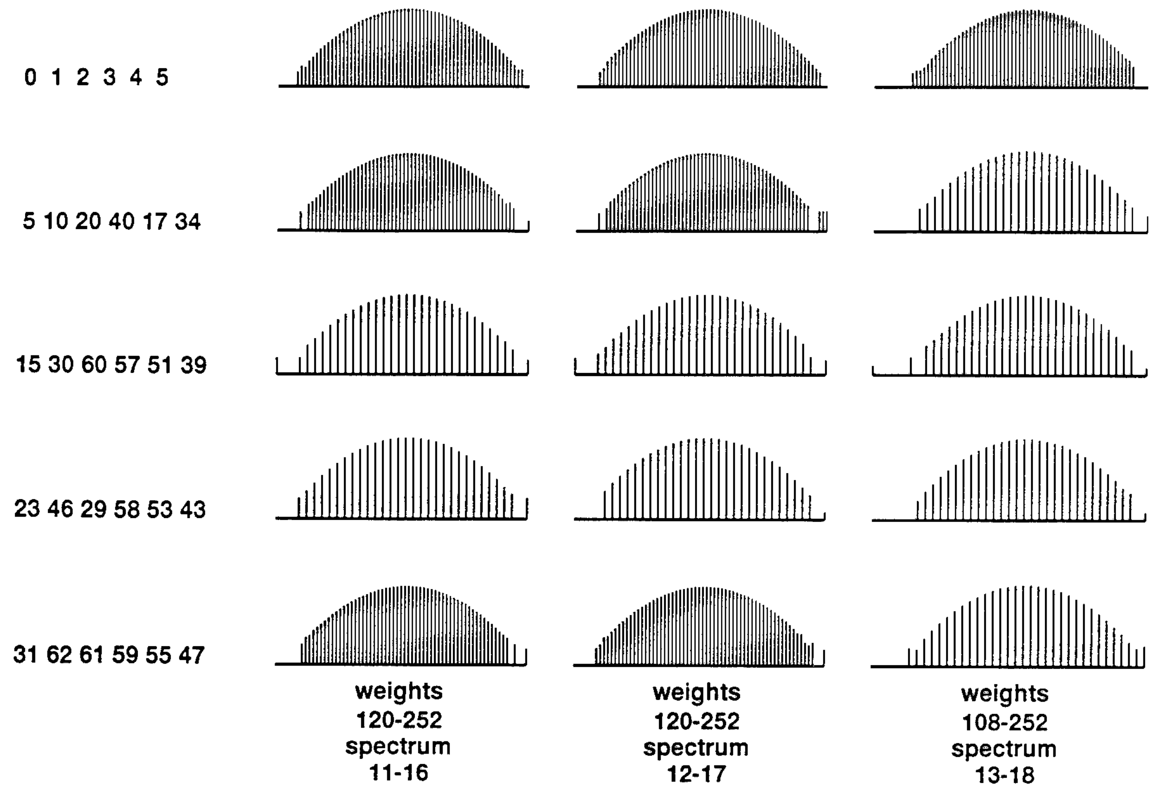

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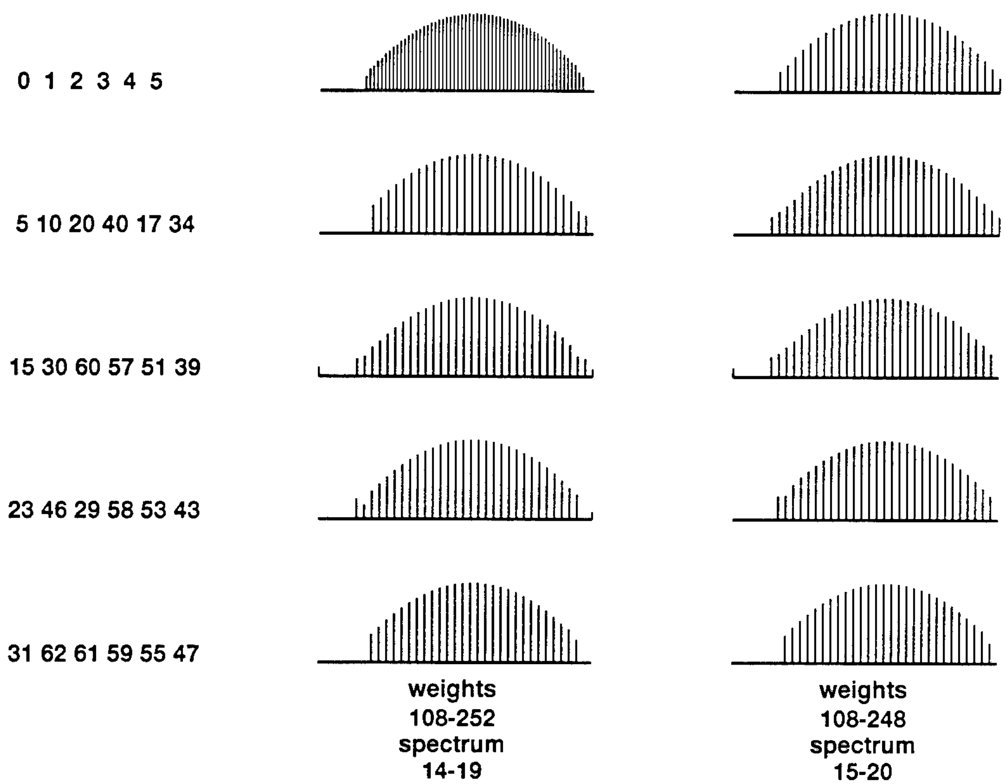
weights 108-252 spectrum 13-18



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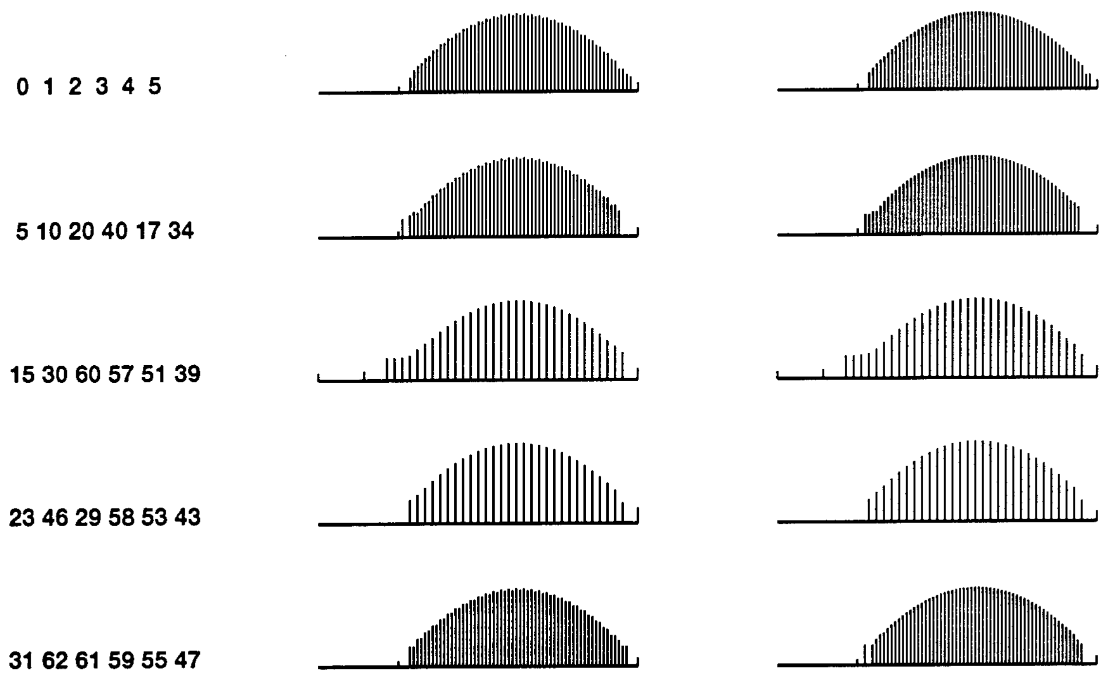
weights 108-248 spectrum 15-20



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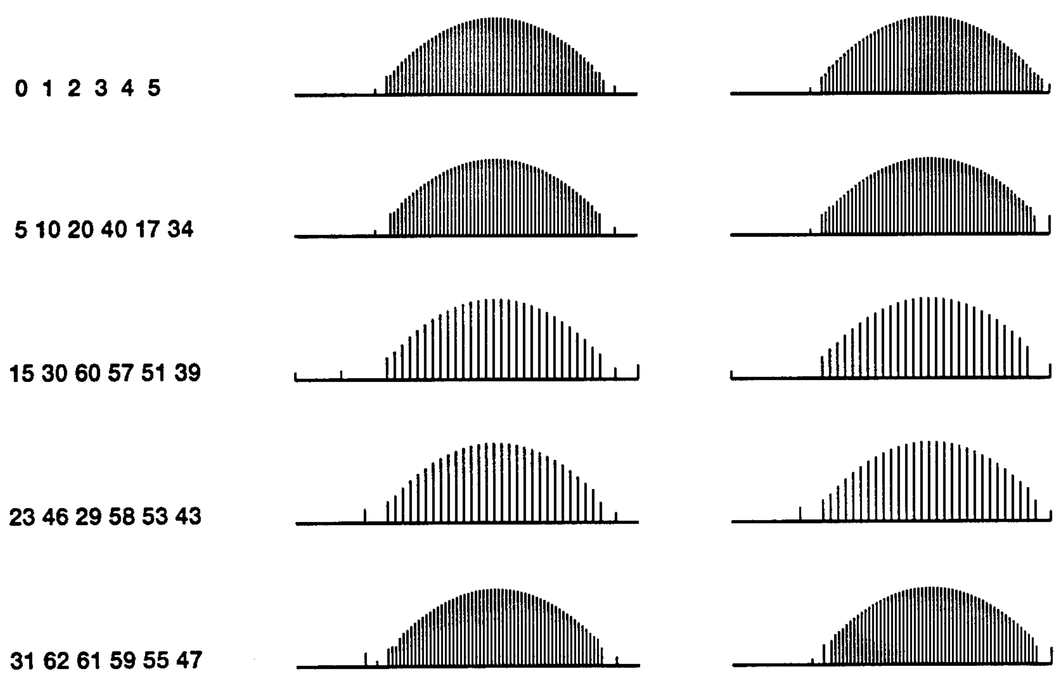
spectrum
16-21

weights
84-252
spectrum
17-22



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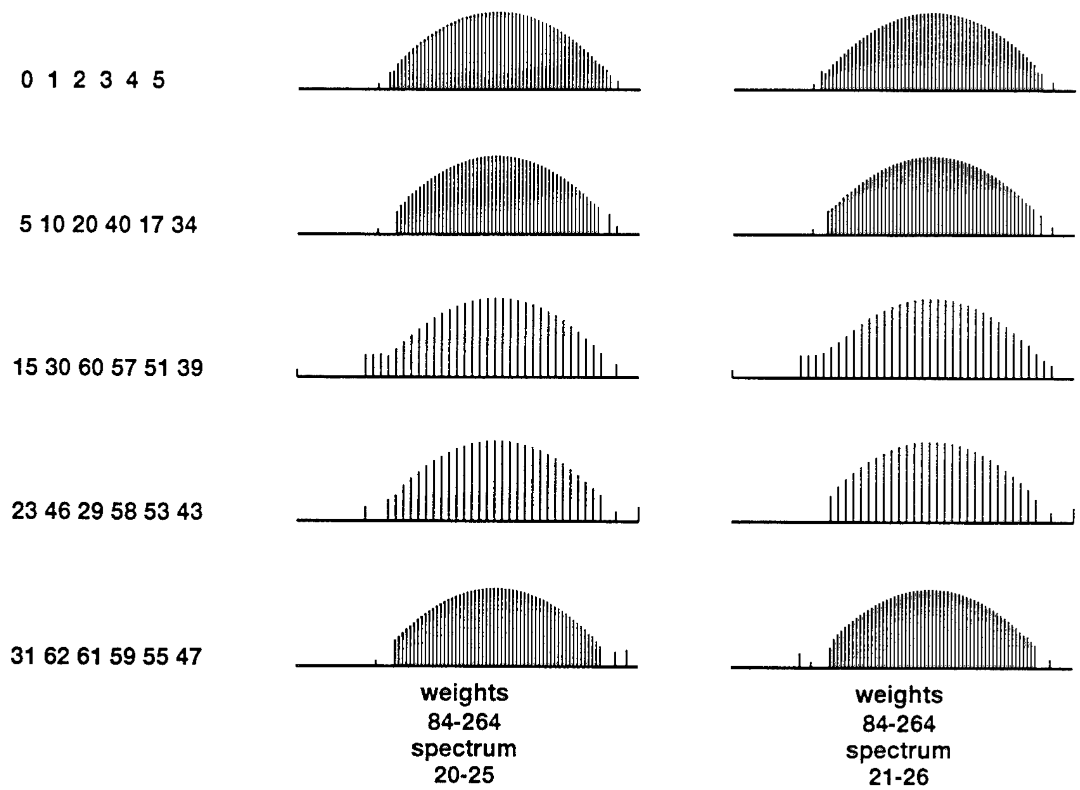
spectrum
18-23

weights
84-252
spectrum
19-24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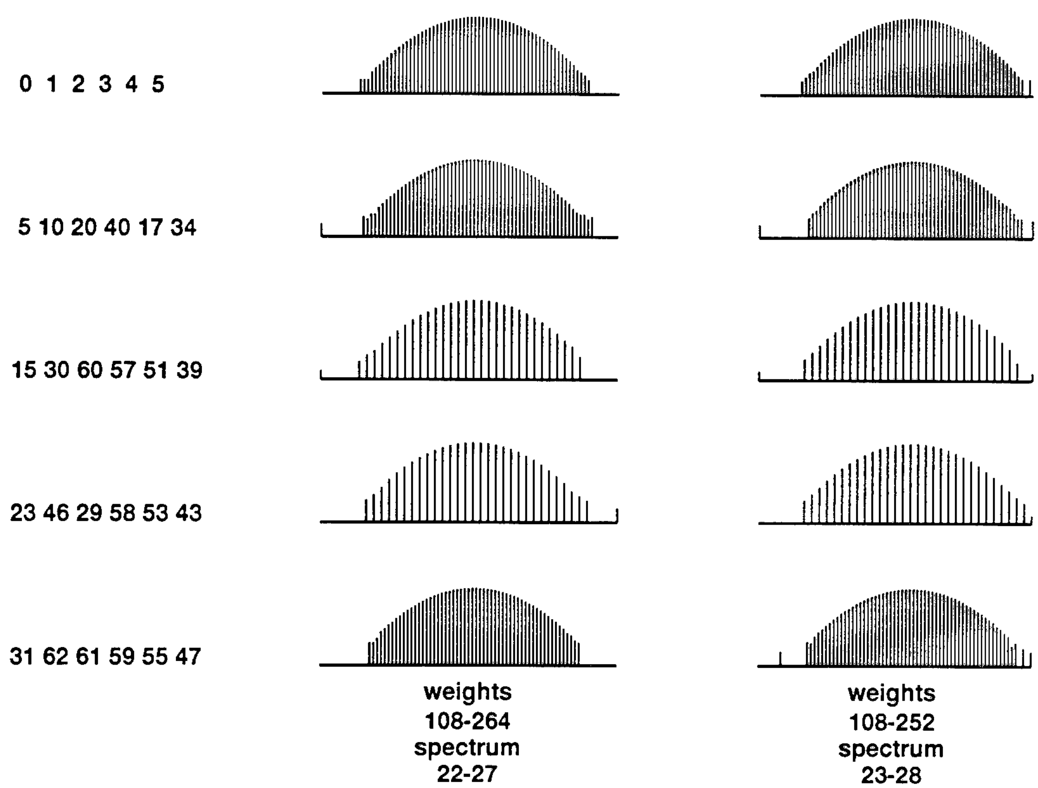
spectrum
20-25
weights
84-264
spectrum
21-26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weights
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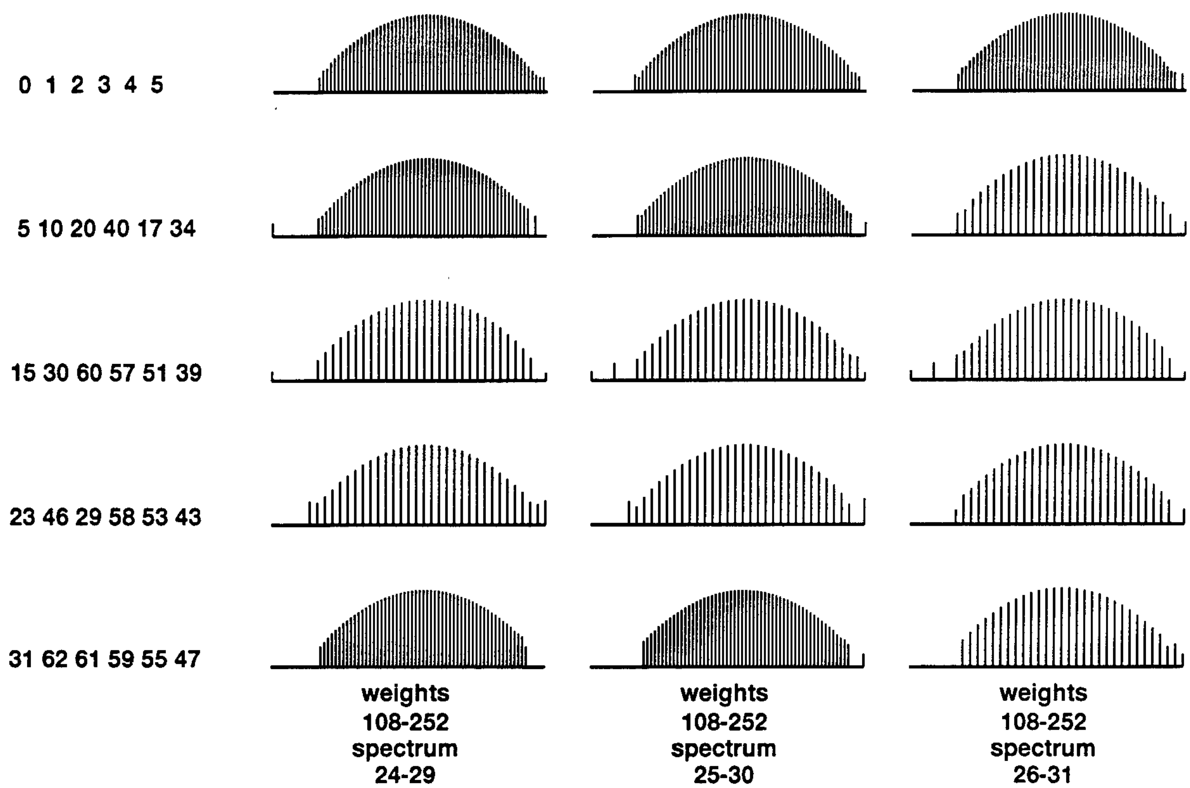
spectrum
23-28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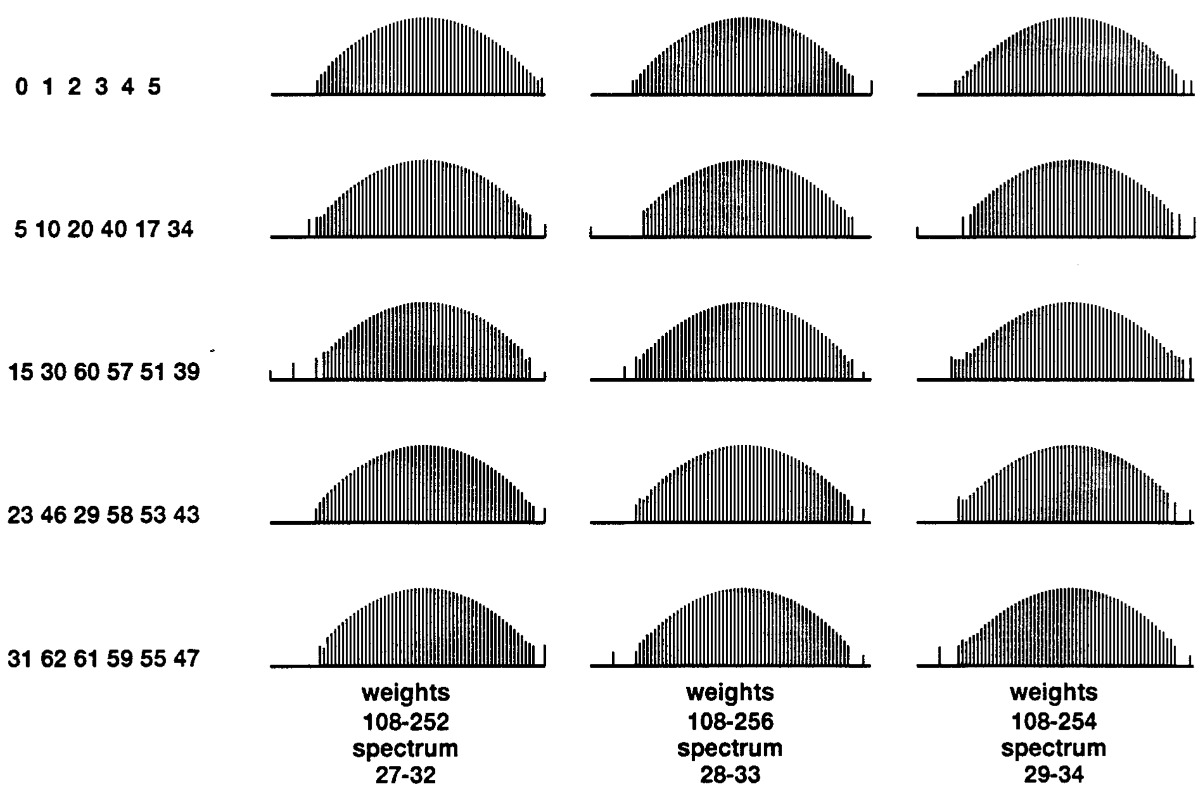
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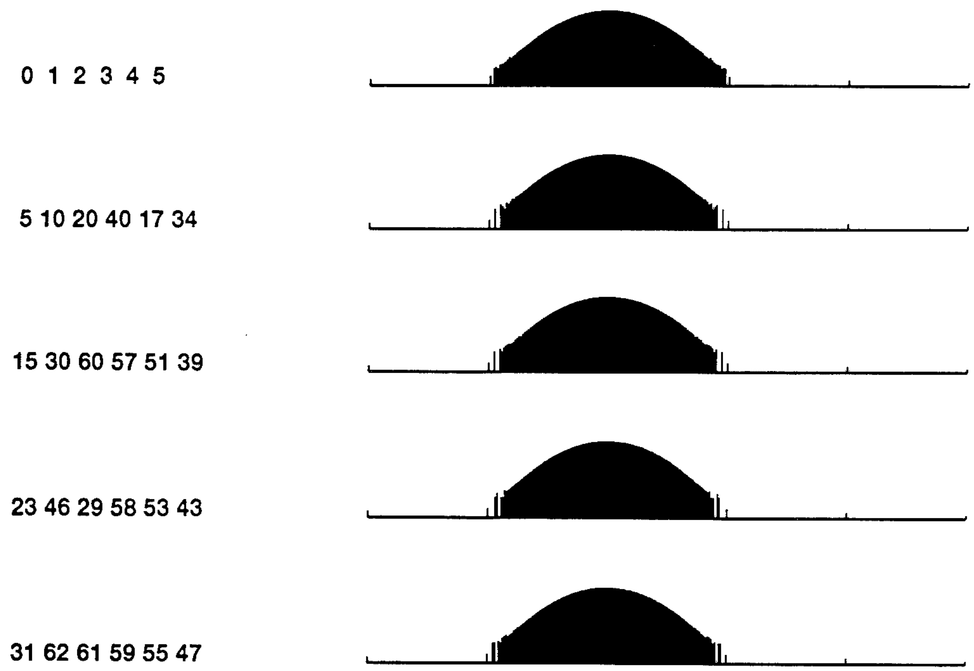
weights 108-254 spectrum 29-34



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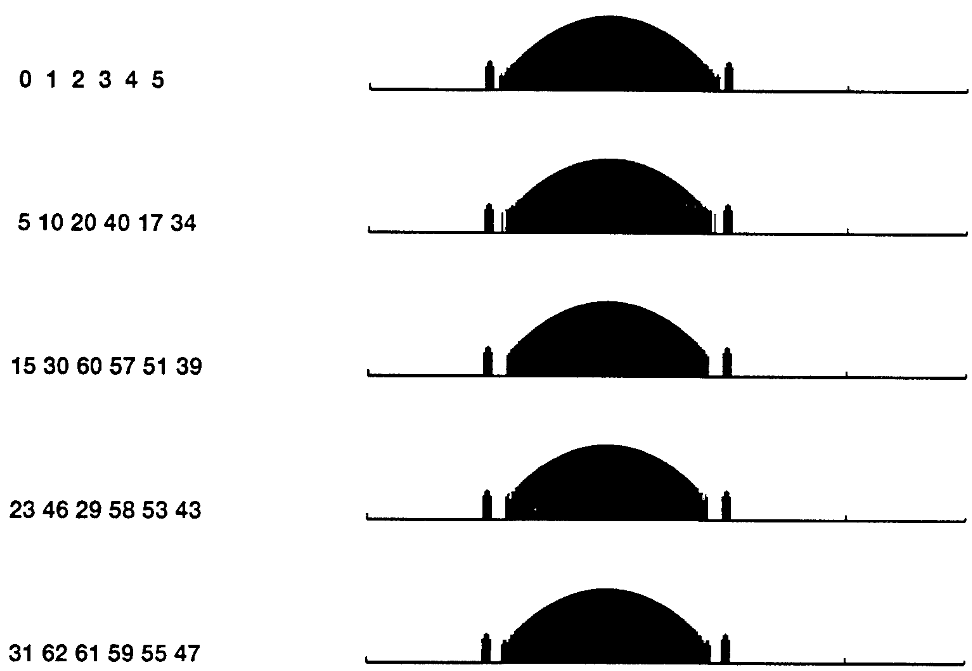
spectrum
61-3



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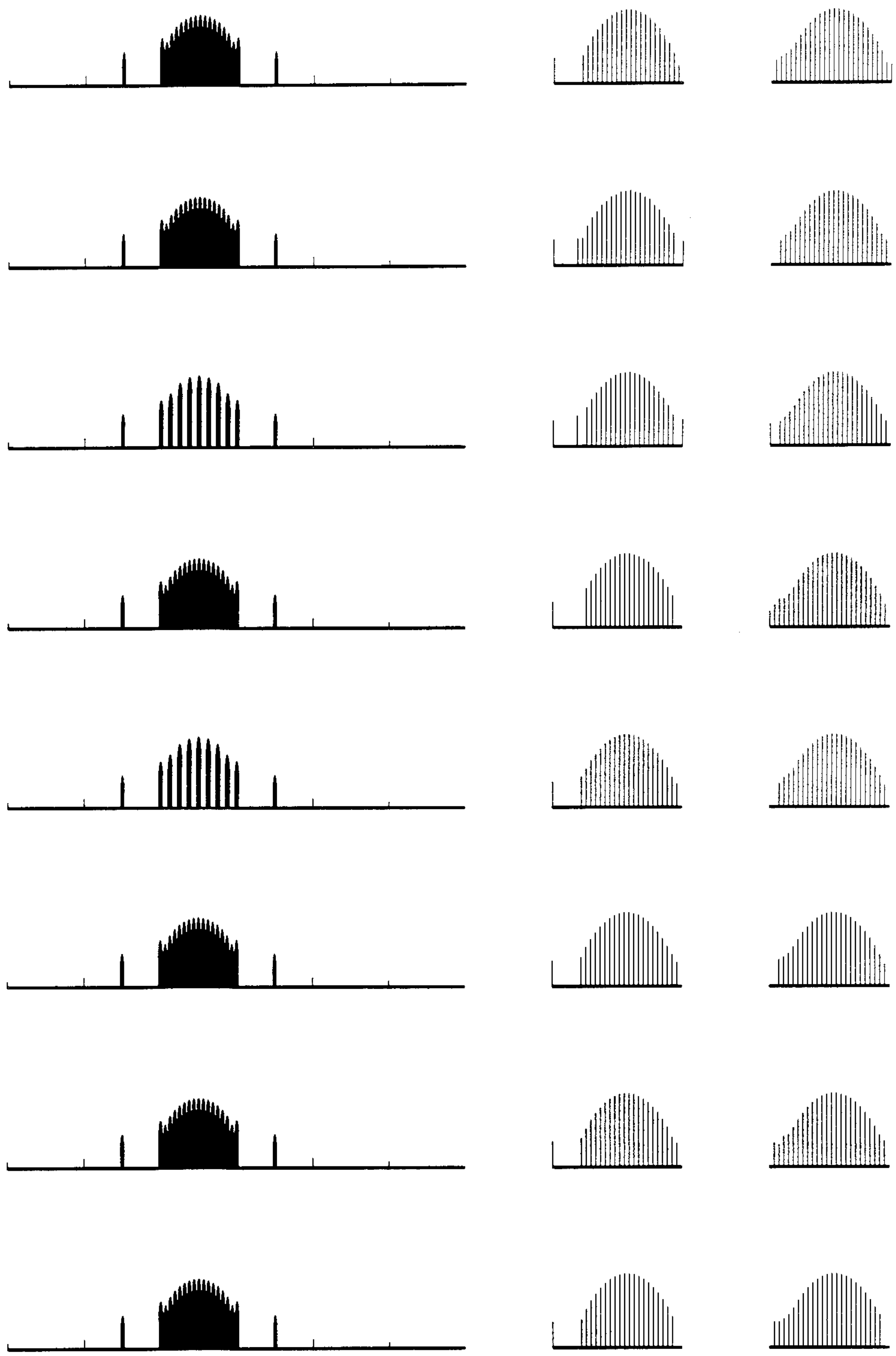
63-378
spectrum
62-4



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82 37 74

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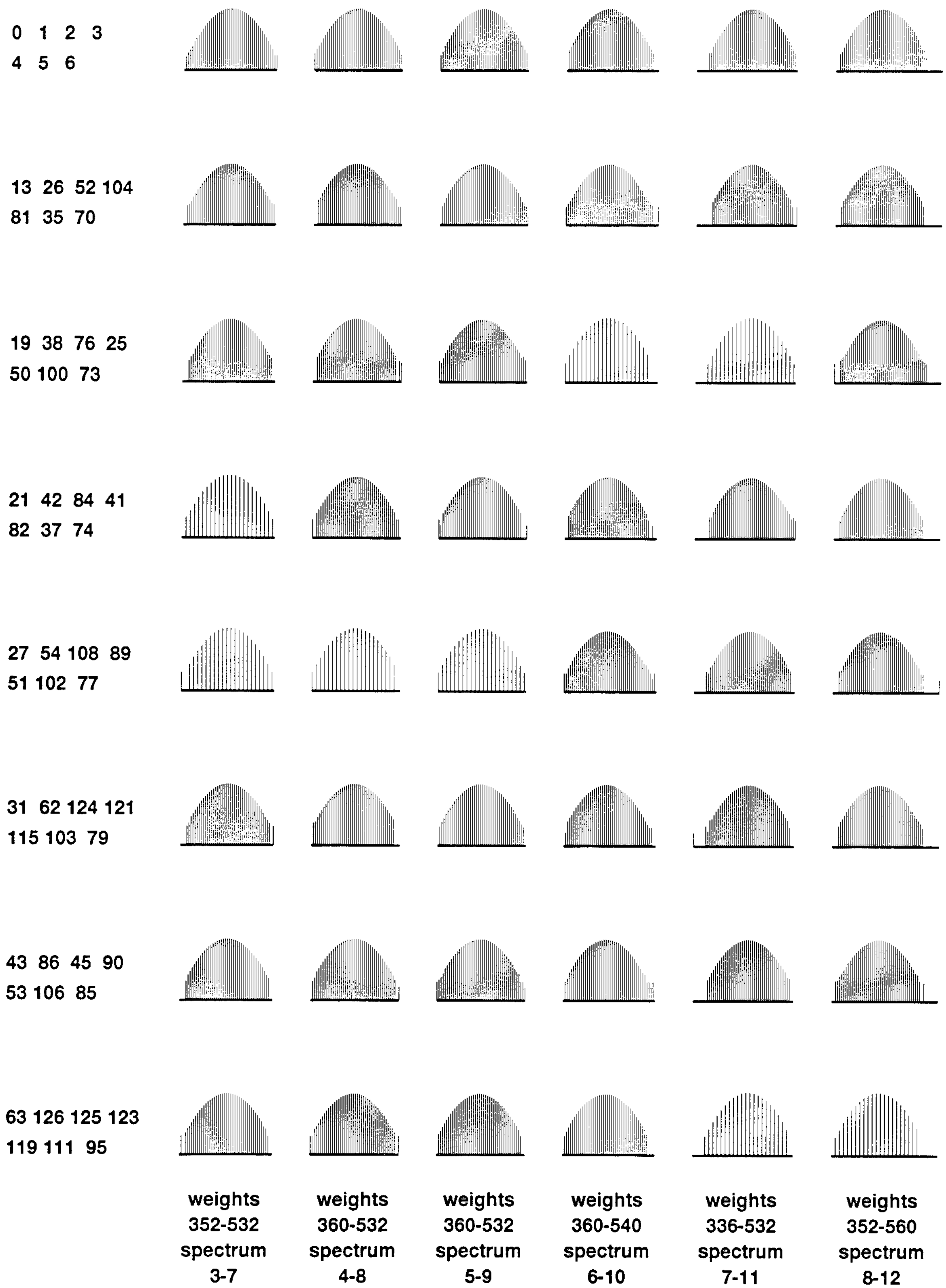
weights 320-536 spectrum 1-5

weights 336-536 spectrum 2-6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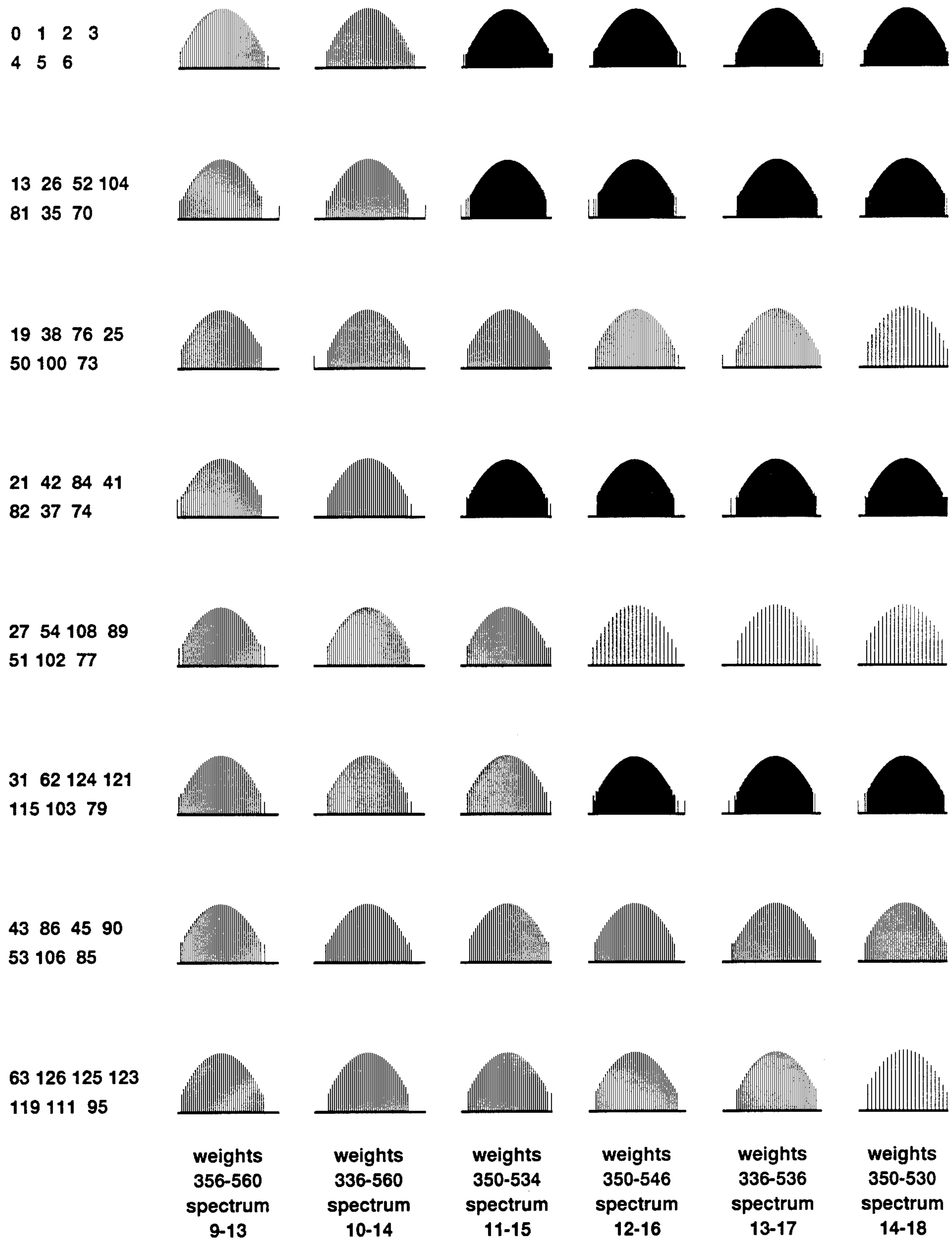
weights 352-560 spectrum 8-12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63 126 125 123
119 111 95
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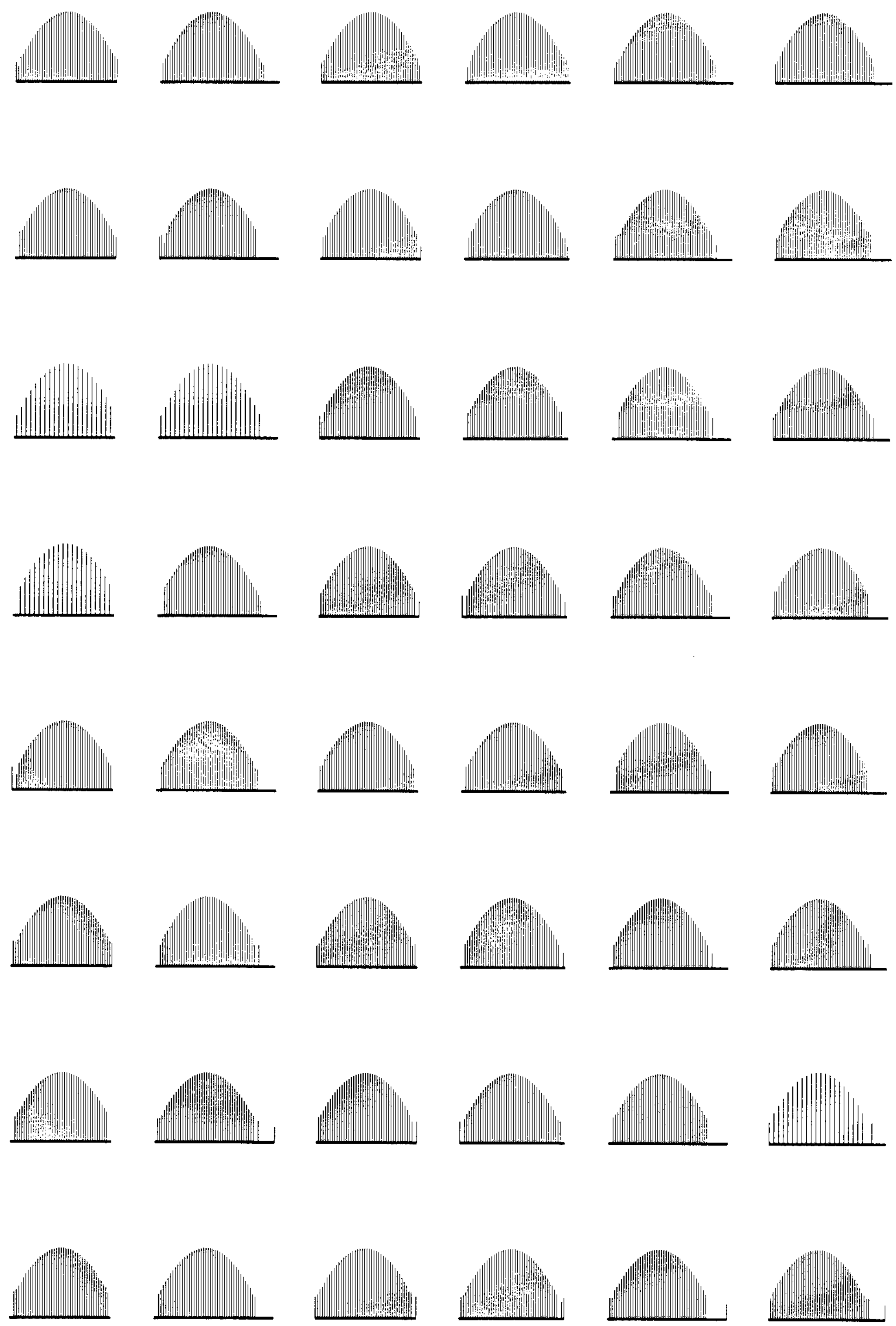
weights 350-546 spectrum 12-16
weights 336-536 spectrum 13-17
weights 350-530 spectrum 14-18



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115 103 79

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63 126 125 123
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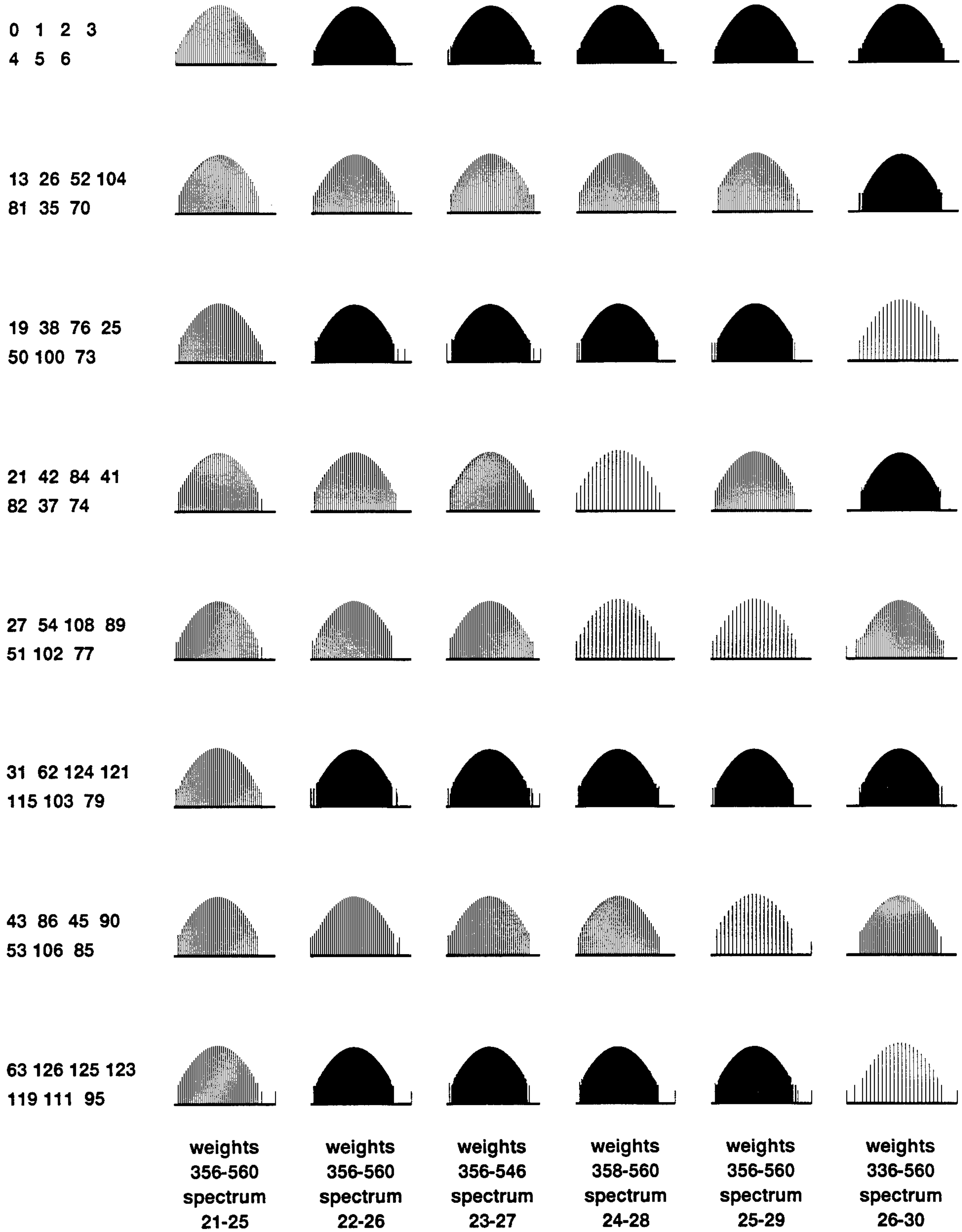
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119 111 95
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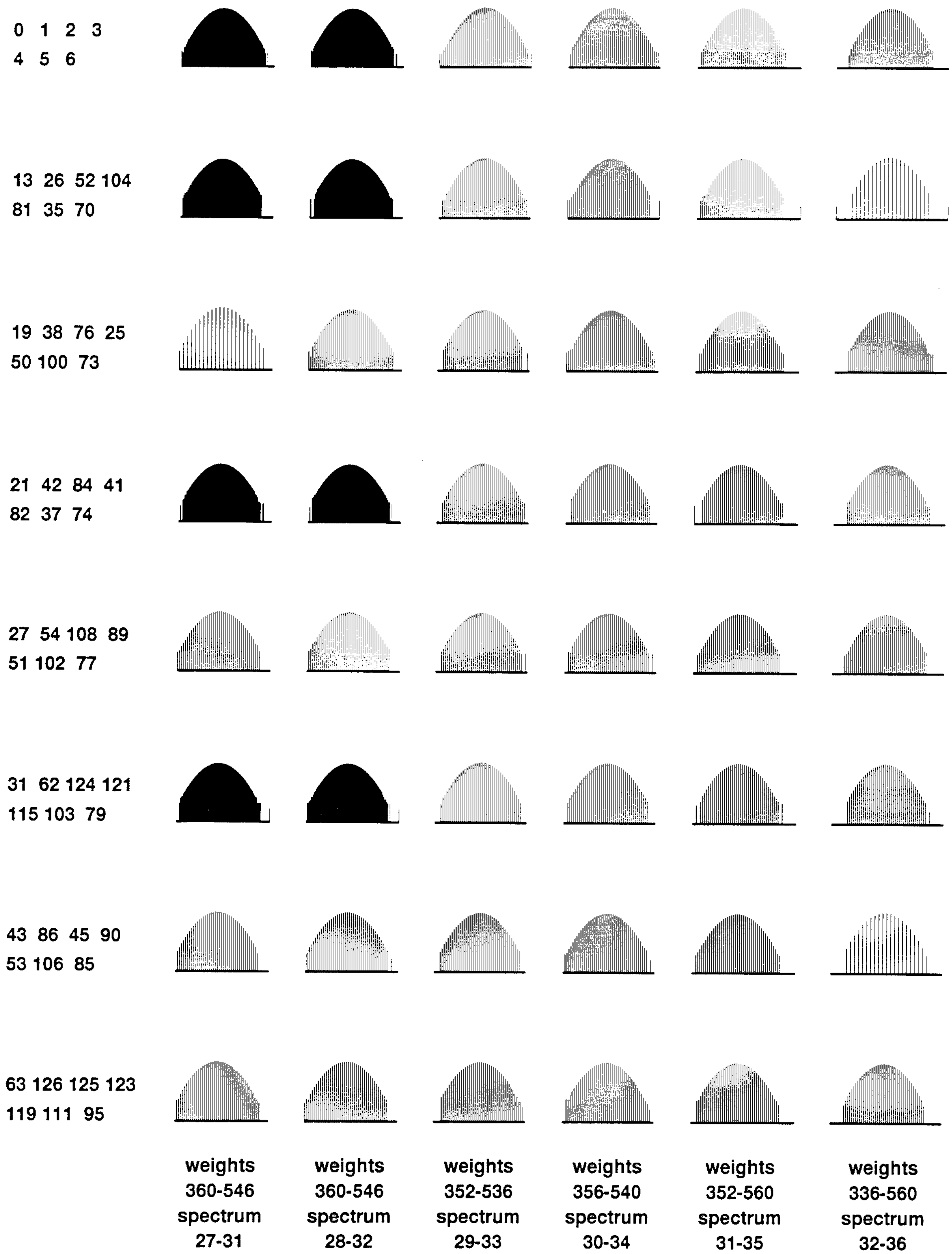
weights 356-560 spectrum 25-29
weights 336-560 spectrum 26-30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119 111 95
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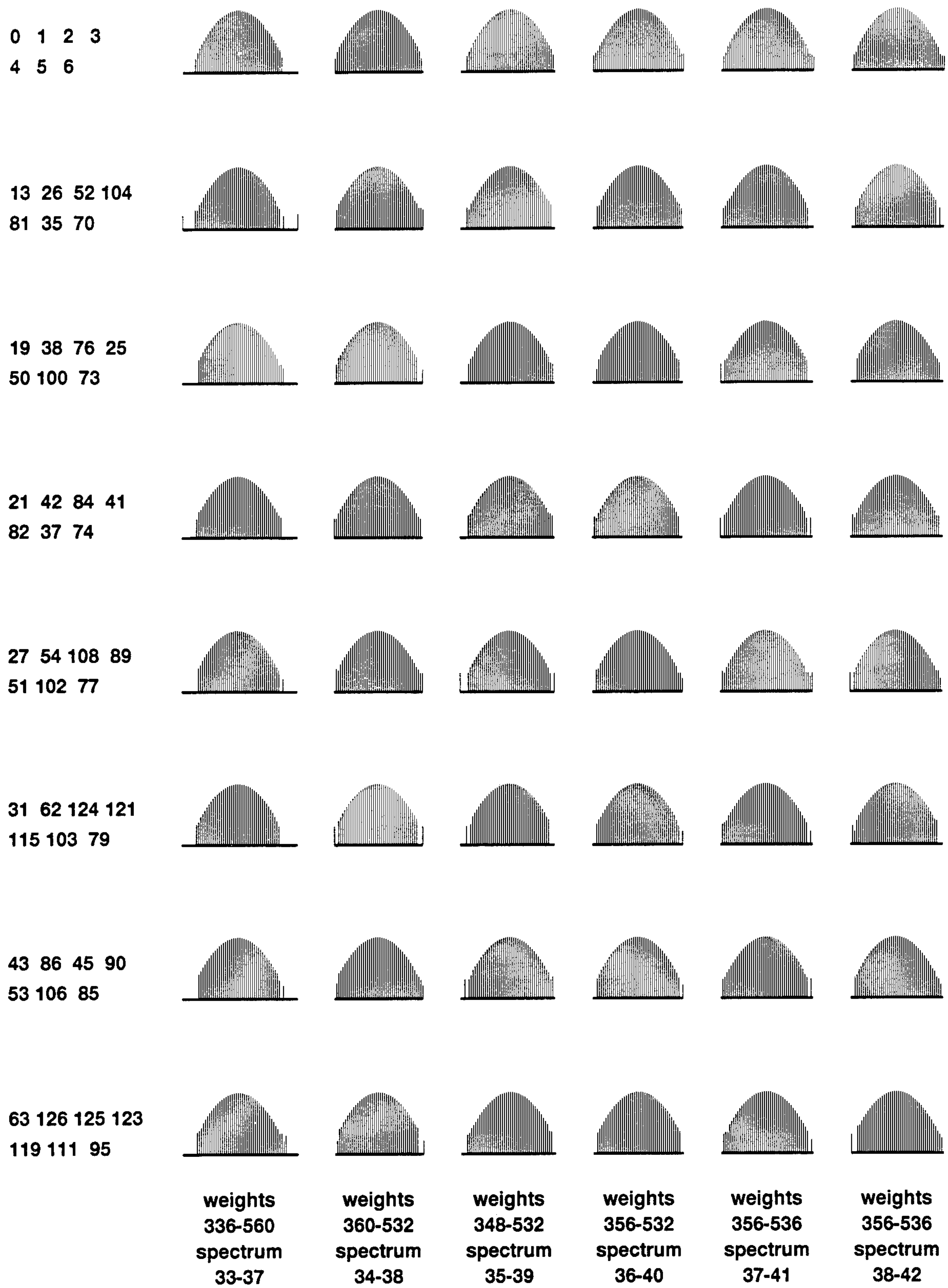
weights 352-560 spectrum 31-35
weights 336-560 spectrum 32-36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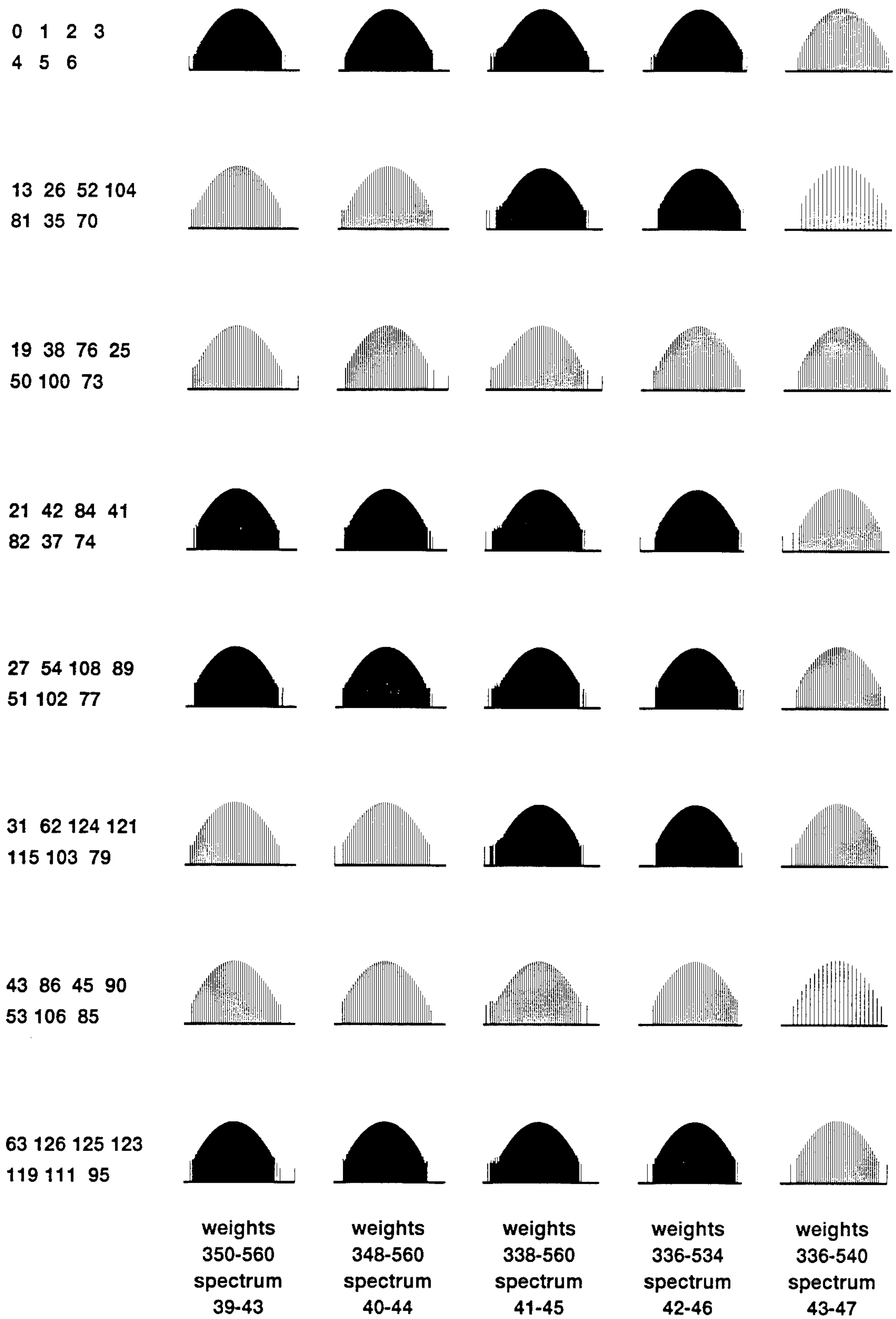
weights 356-532 spectrum 36-40
weights 356-536 spectrum 37-41
weights 356-536 spectrum 38-42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weights 336-534 spectrum 42-46
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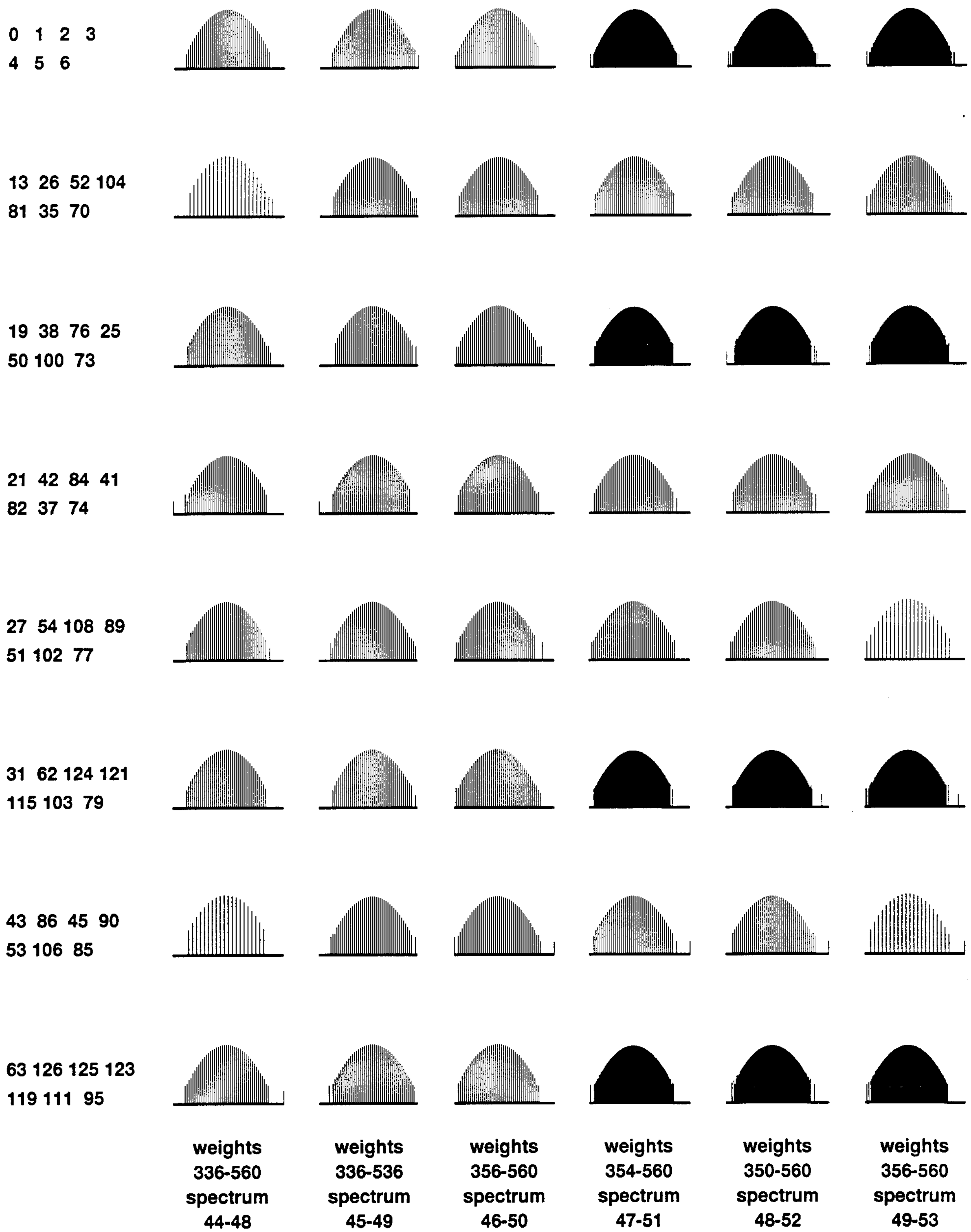
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119 111 95
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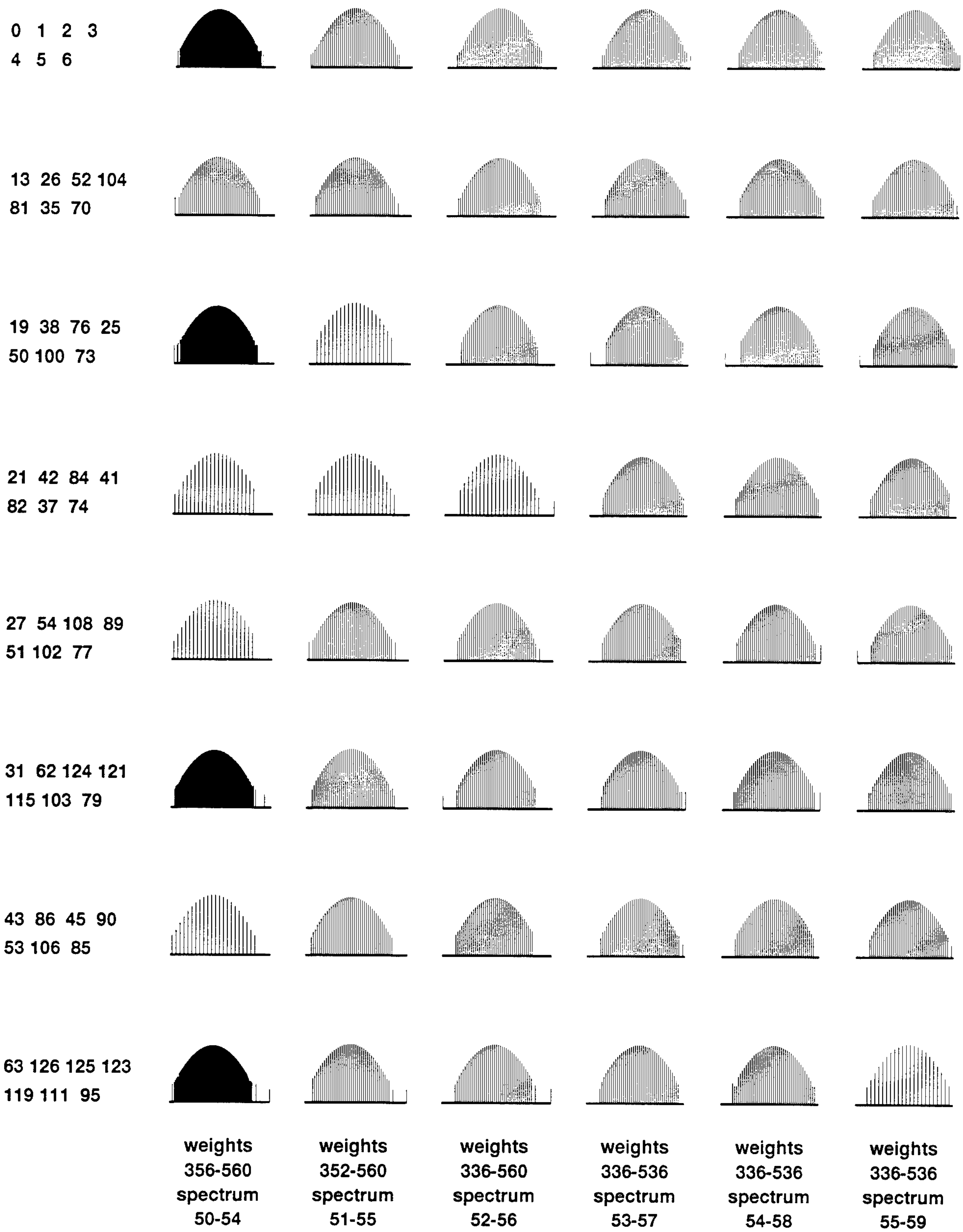
weights 350-560 spectrum 48-52
weights 356-560 spectrum 49-53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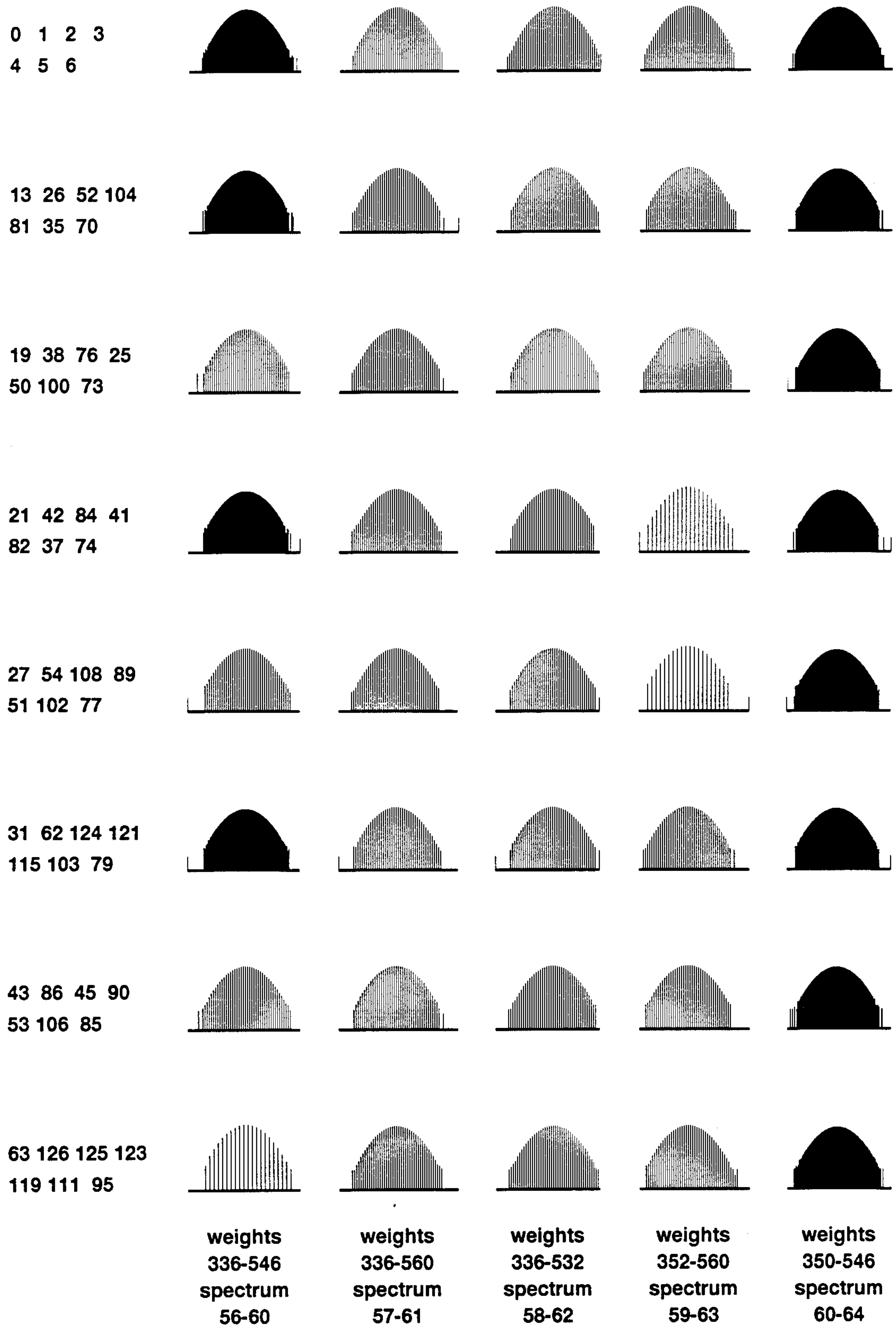
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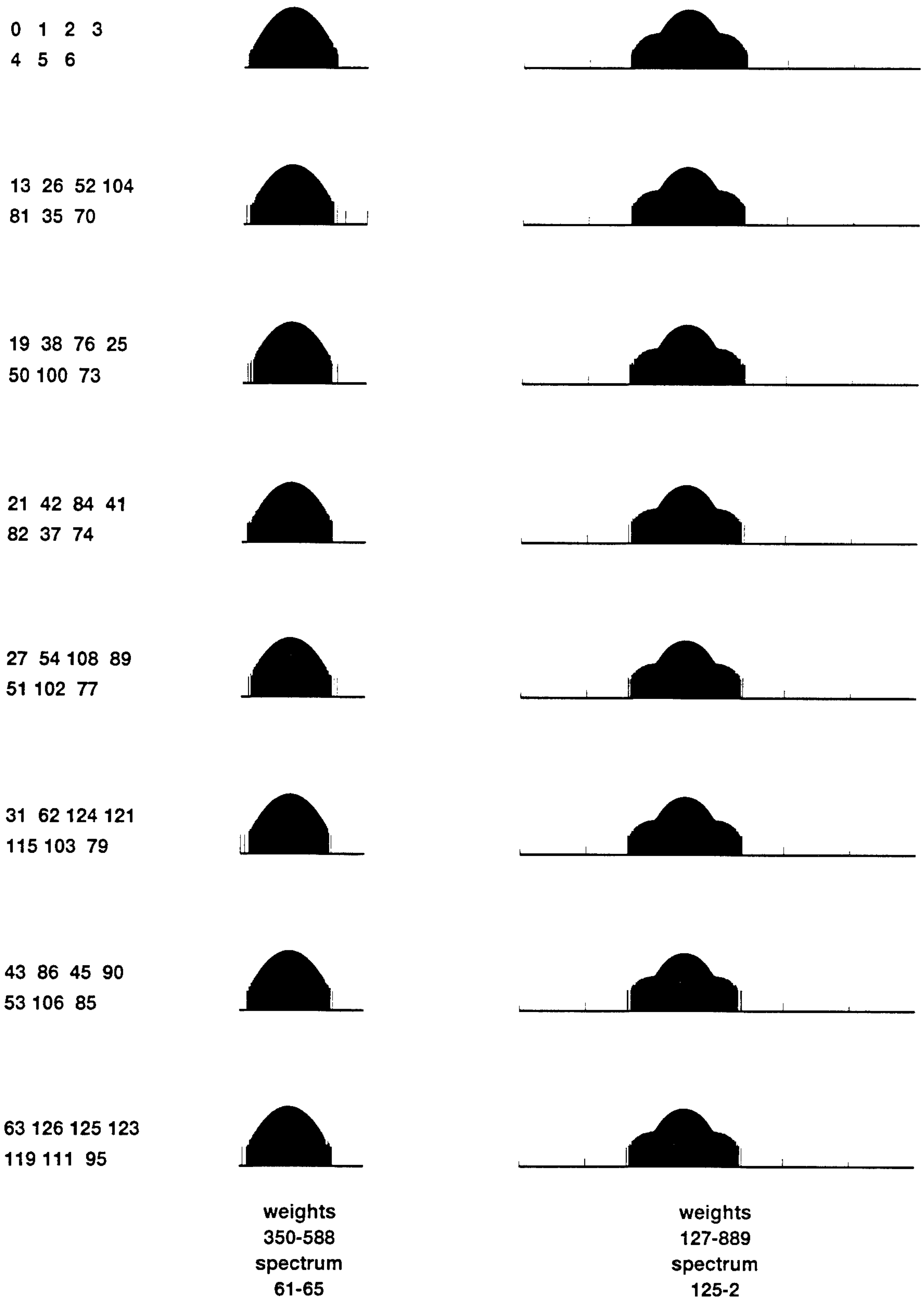
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spectrum
61-65
weights
127-889
spectrum
125-2

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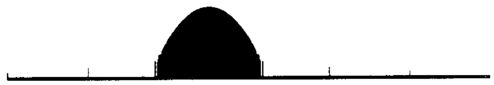

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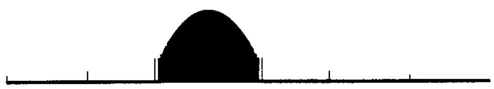

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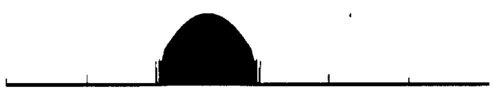

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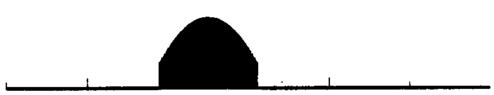

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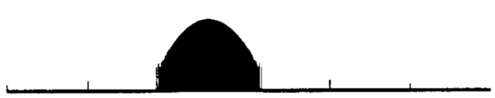

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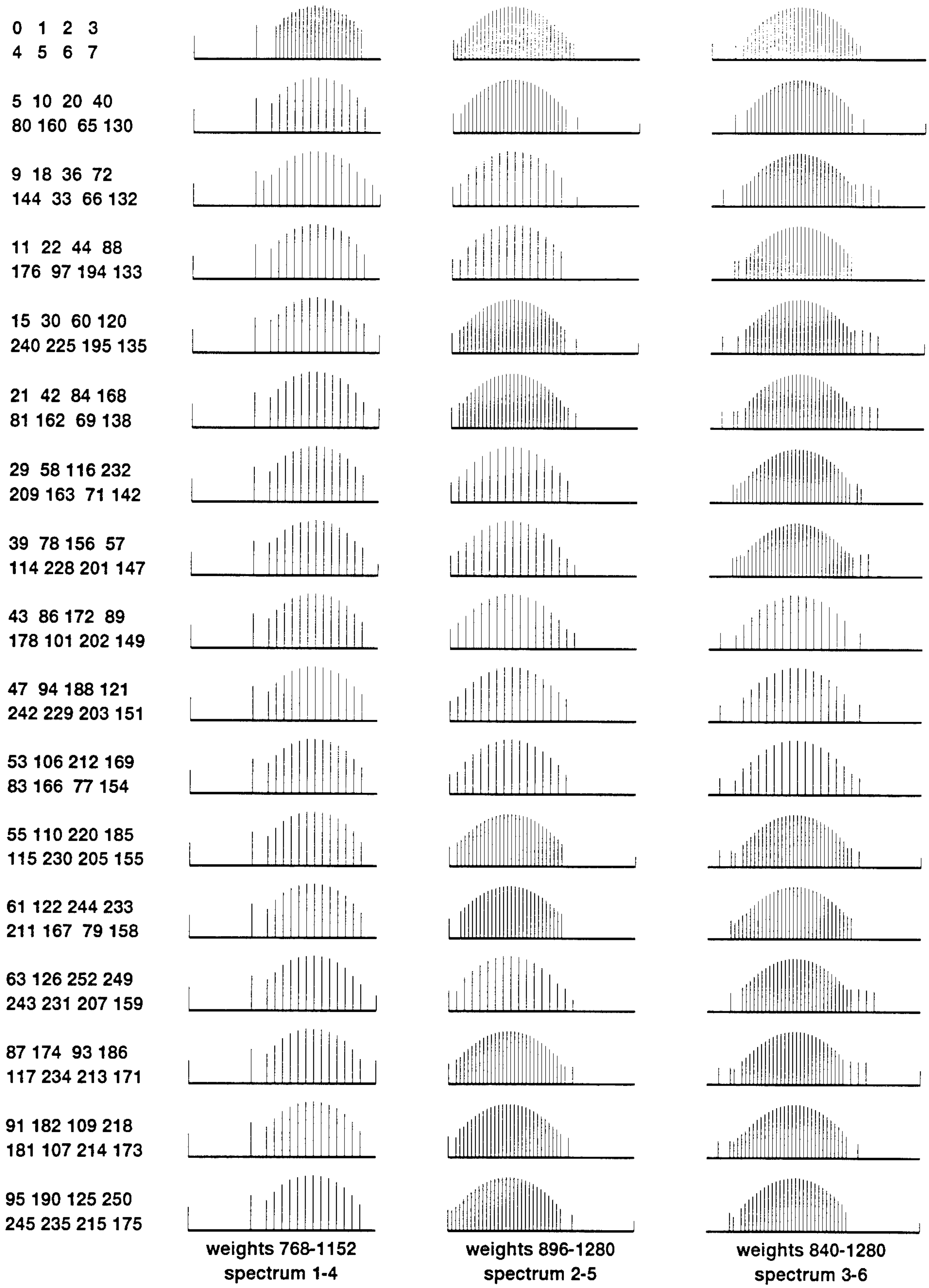
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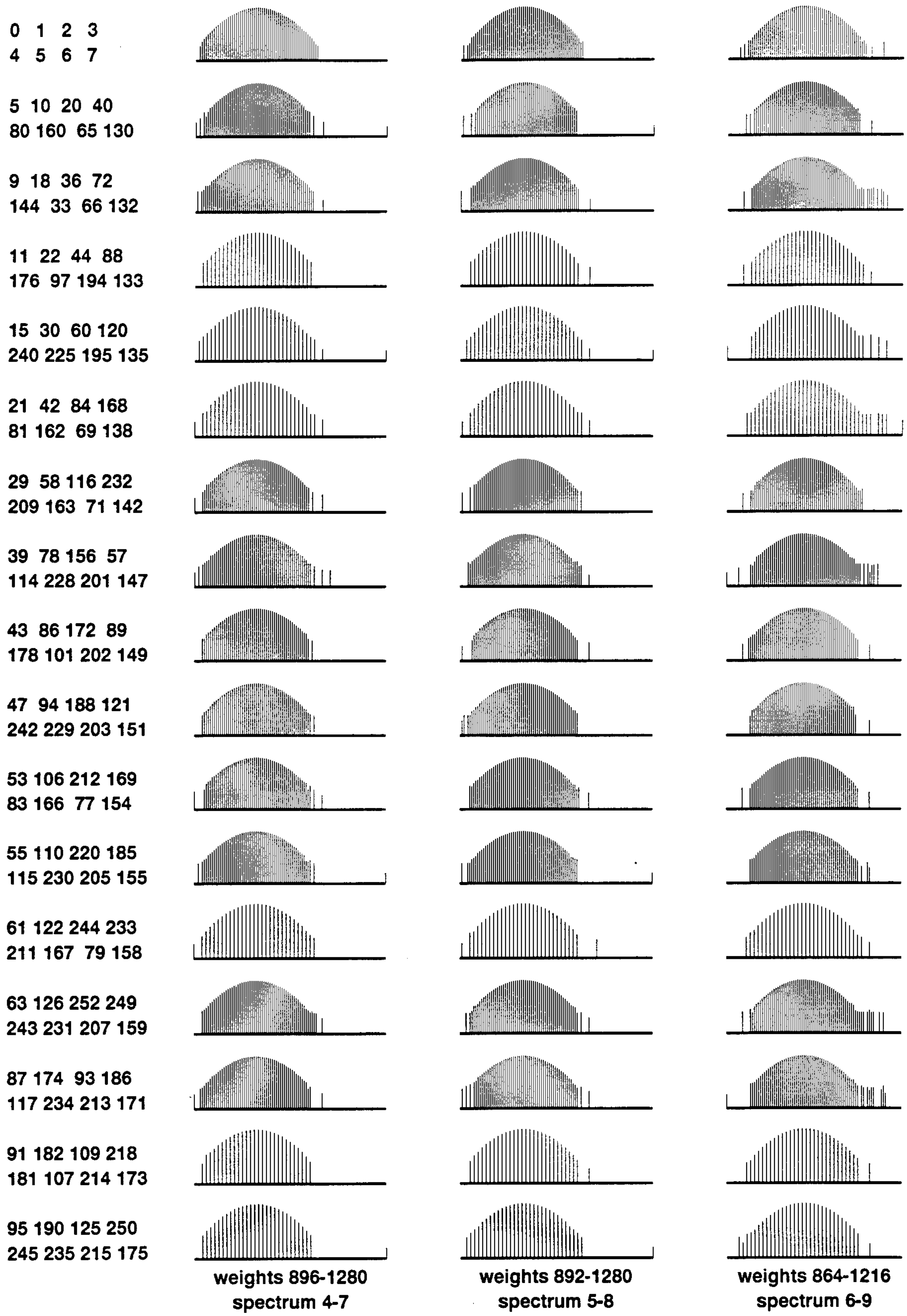
spectrum 3-6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63 126 252 249
243 231 207 159
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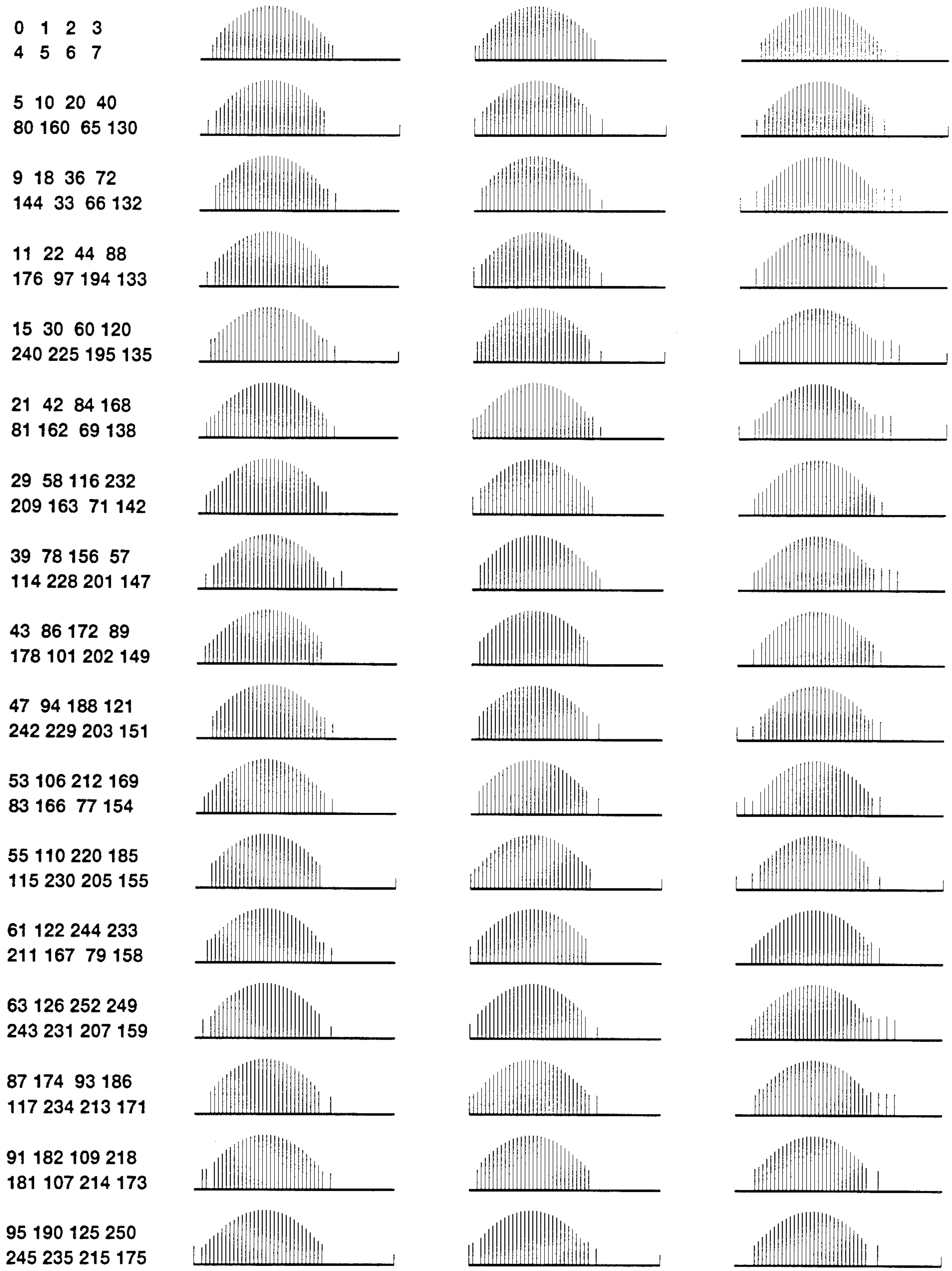
spectrum 6-9



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spectrum 8-11

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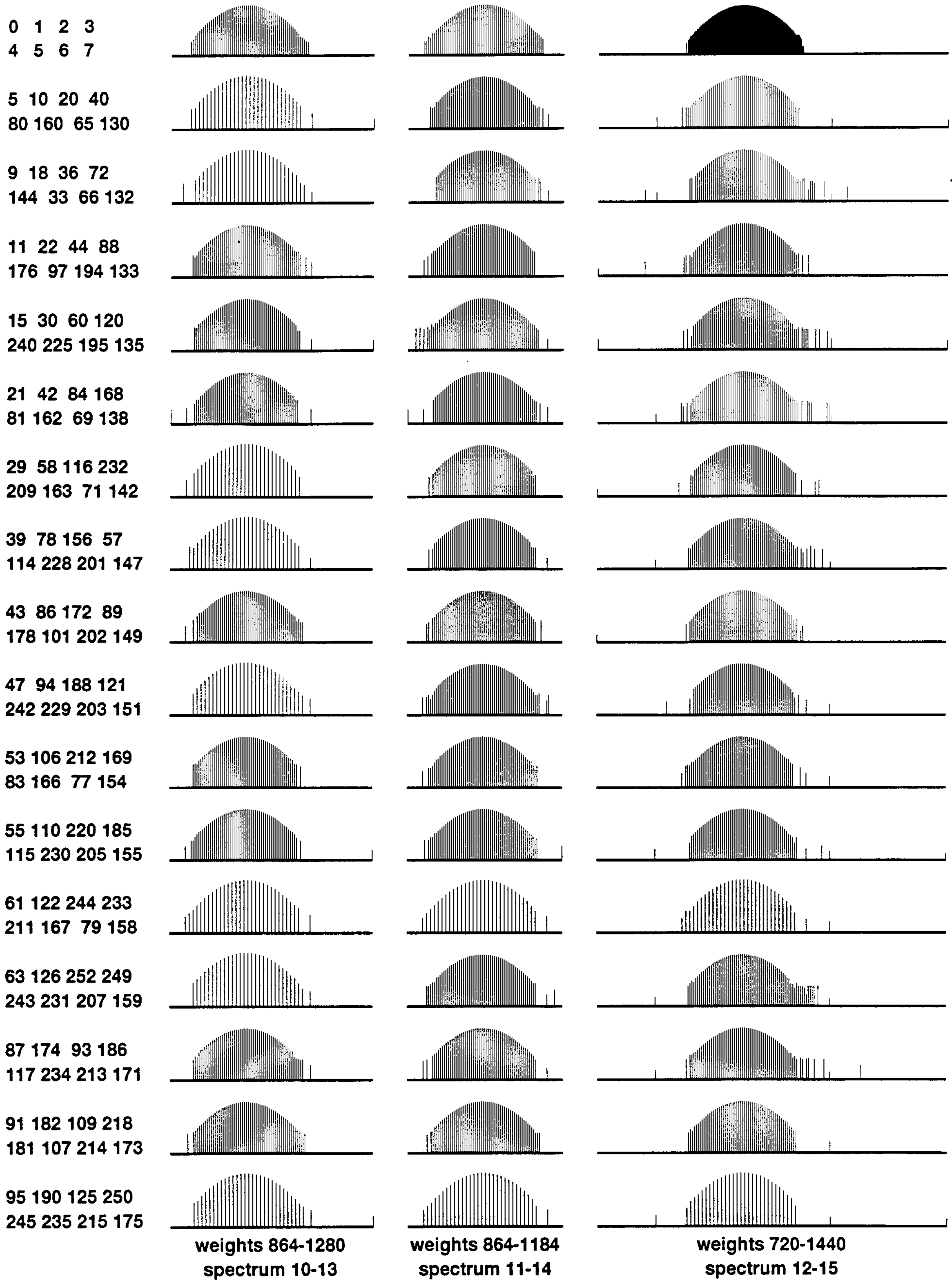
spectrum 9-12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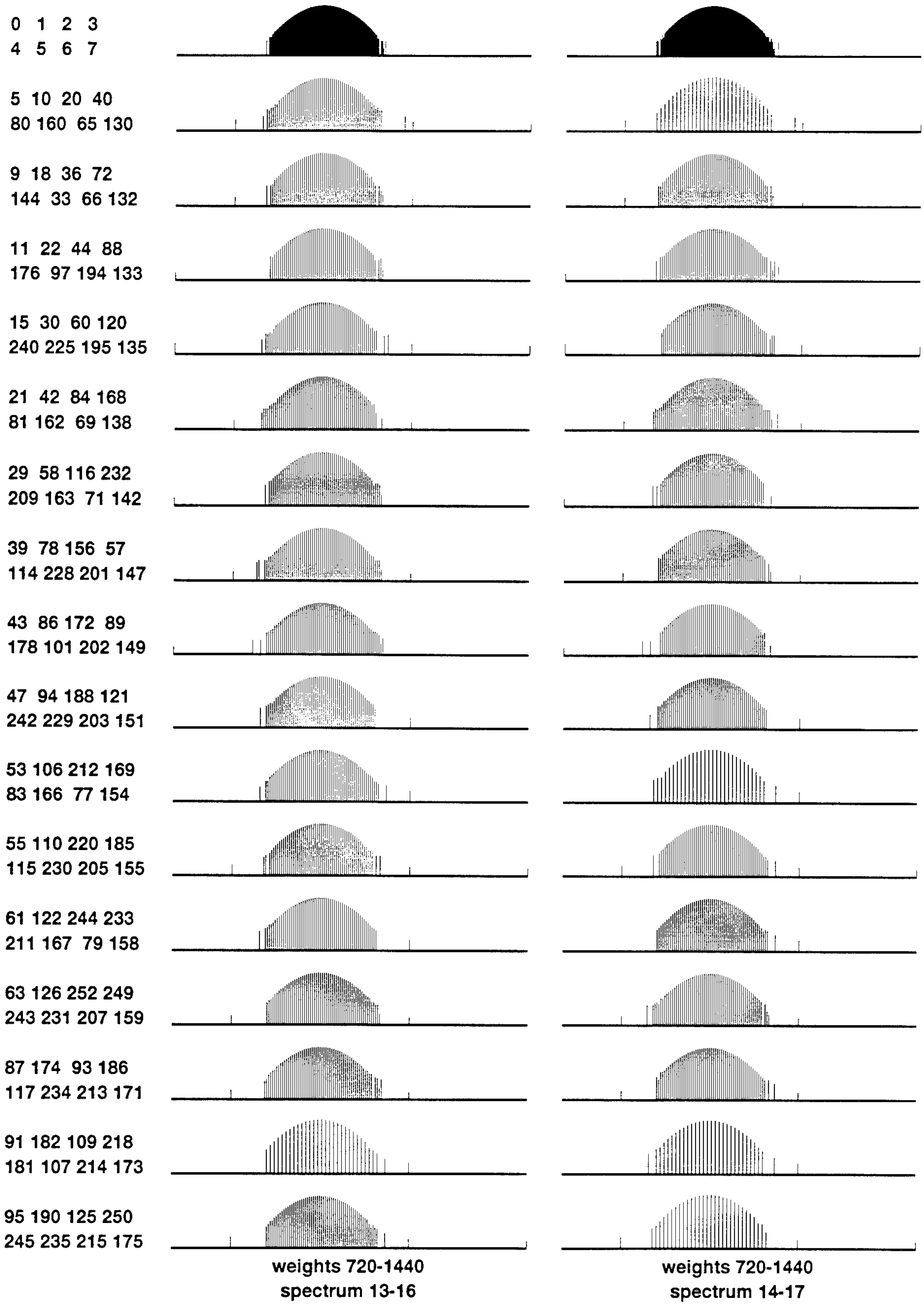
weights 864-1184
spectrum 11-14
weights 720-1440
spectrum 12-15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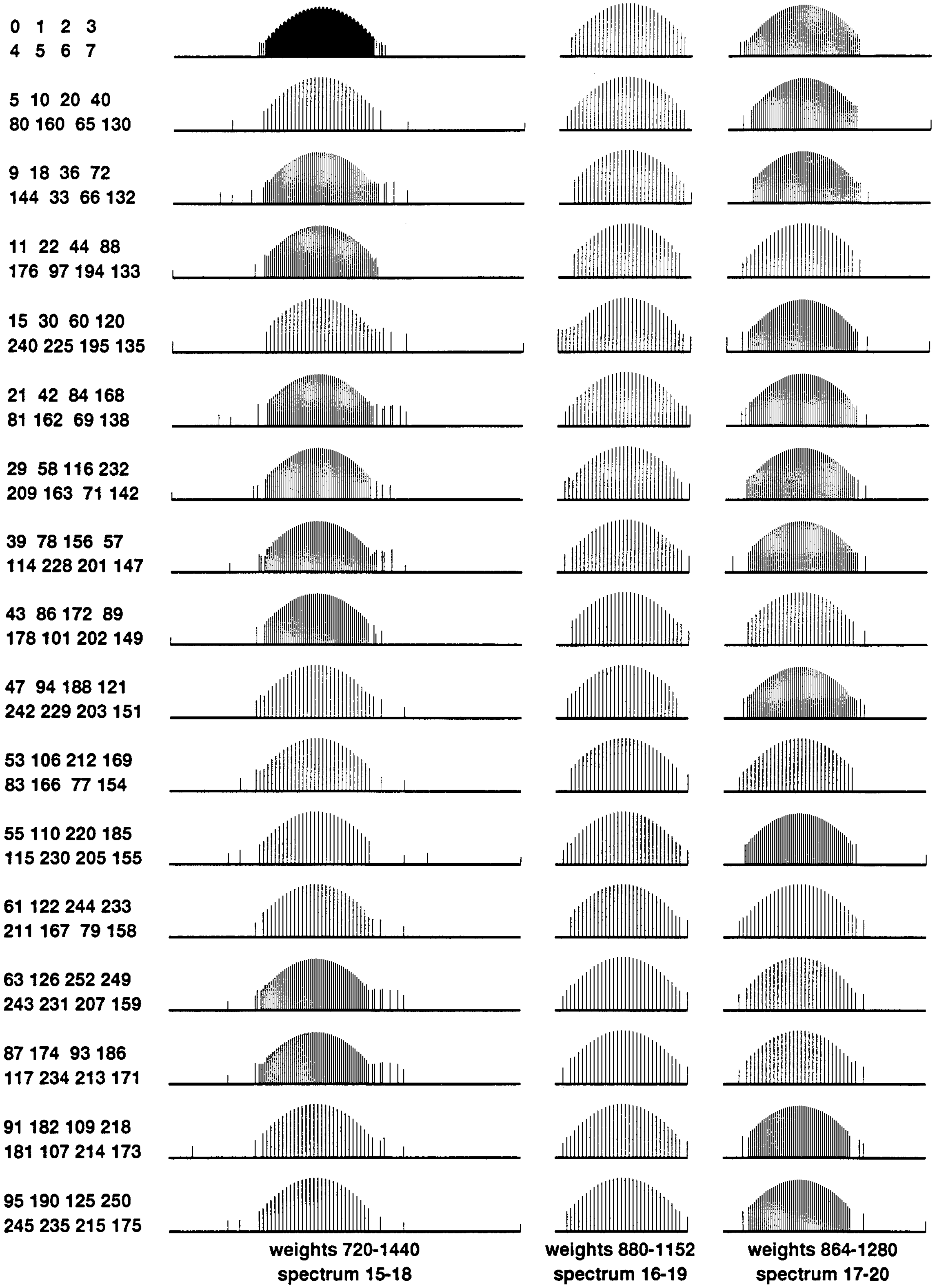
weights 720-1440
spectrum 14-17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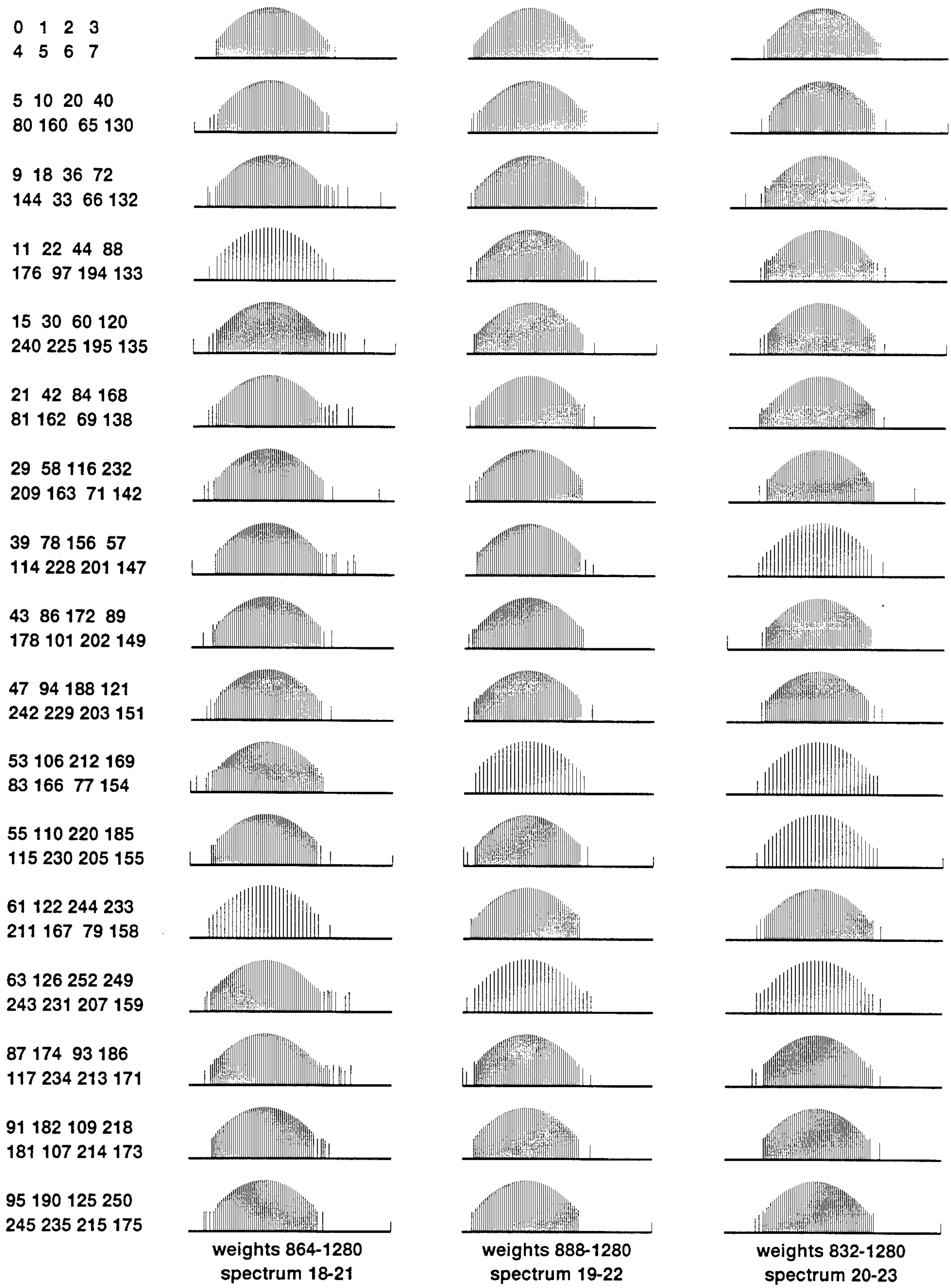
spectrum 17-20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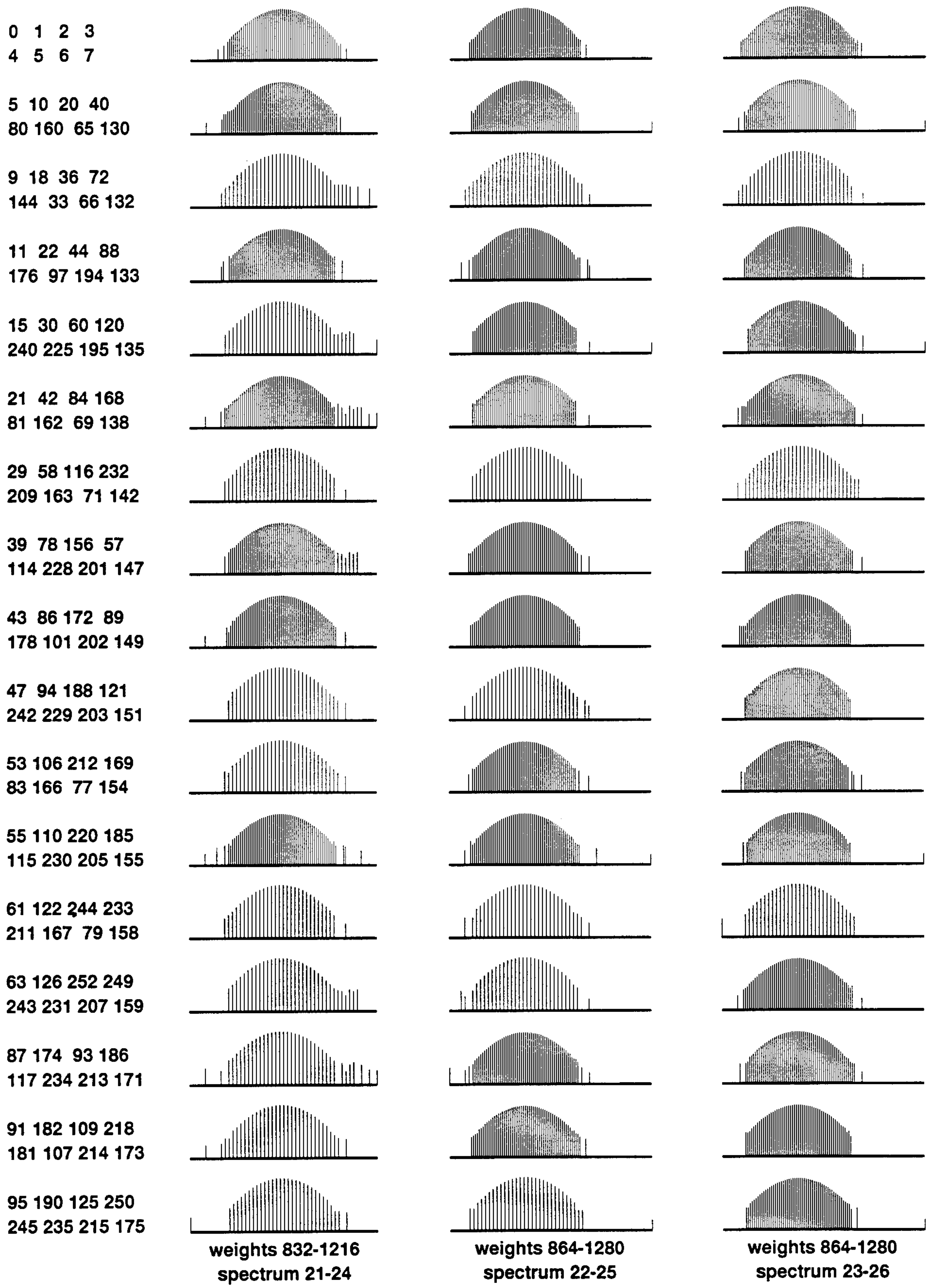
spectrum 18-21
weights 888-1280
spectrum 19-22
weights 832-1280
spectrum 20-23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spectrum 22-25
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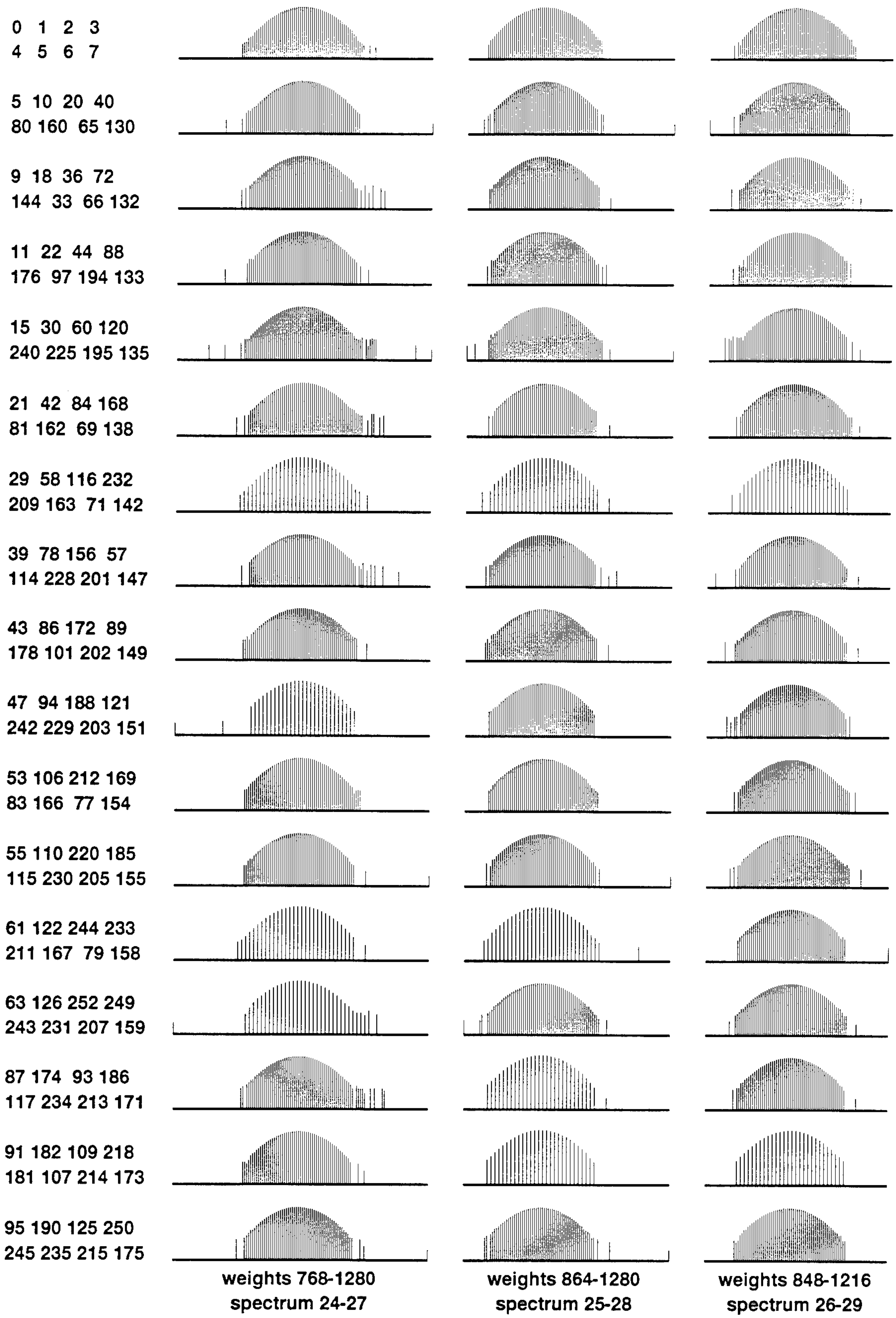
spectrum 23-26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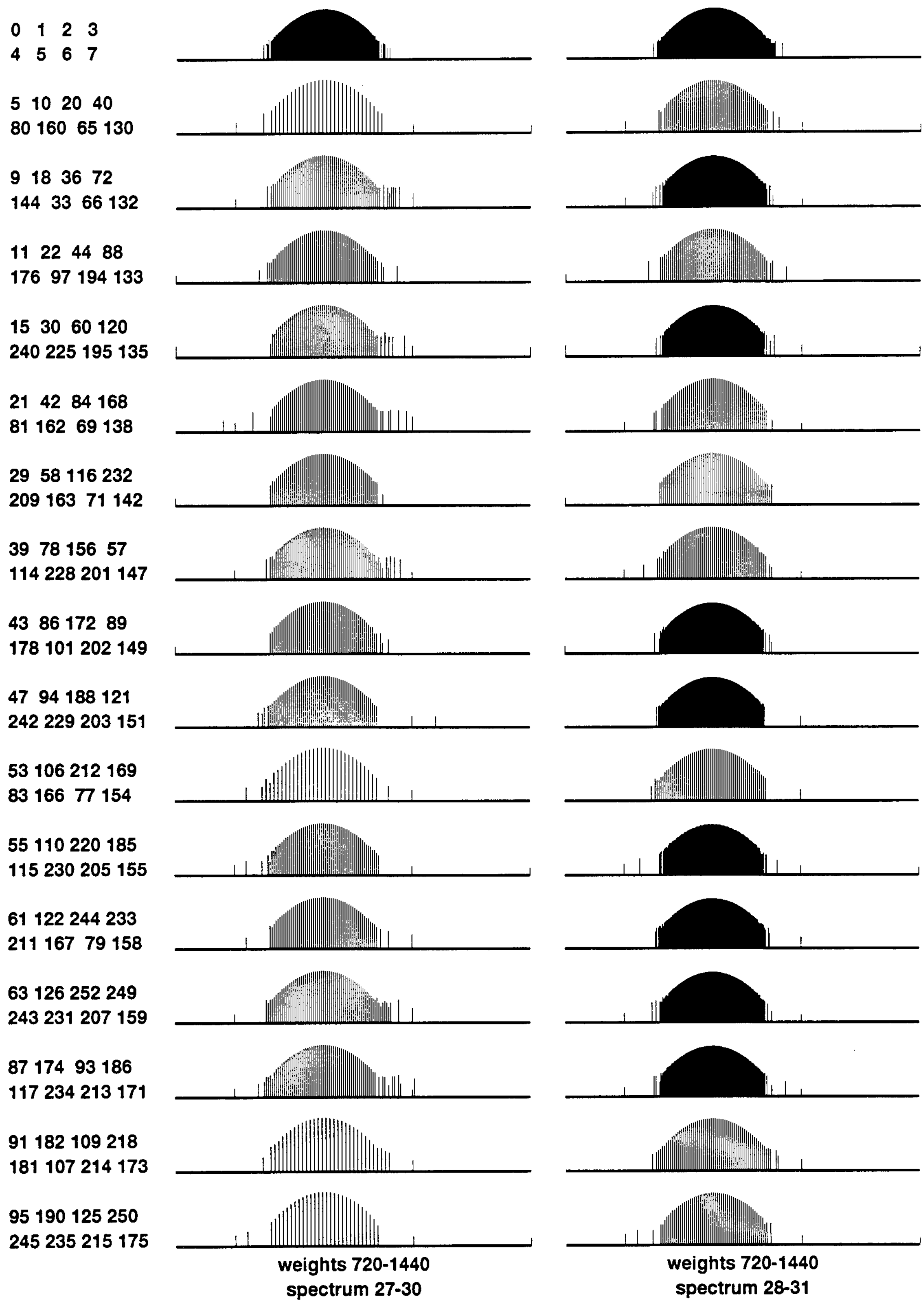
spectrum 25-28
weights 848-1216
spectrum 26-29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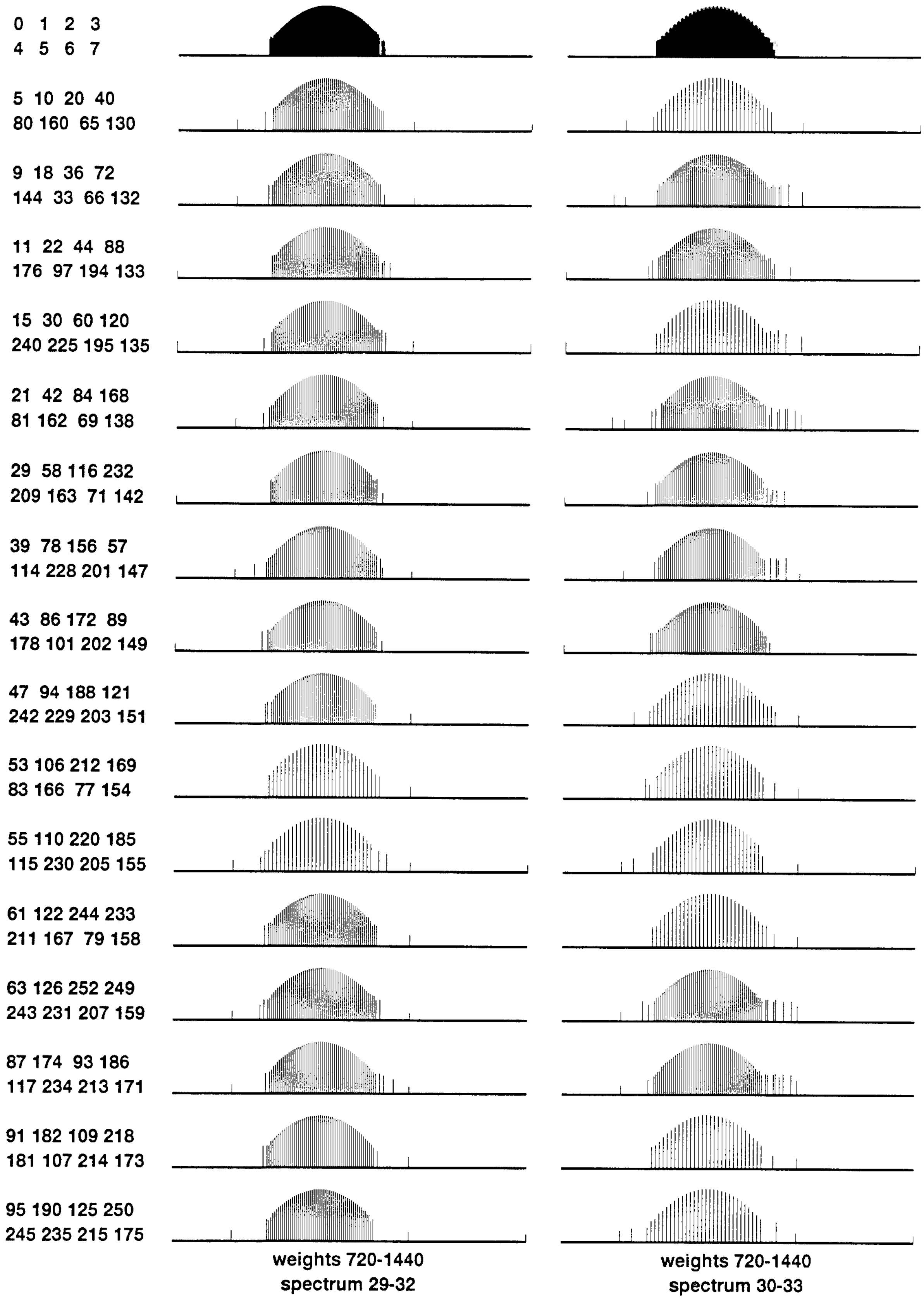
spectrum 28-31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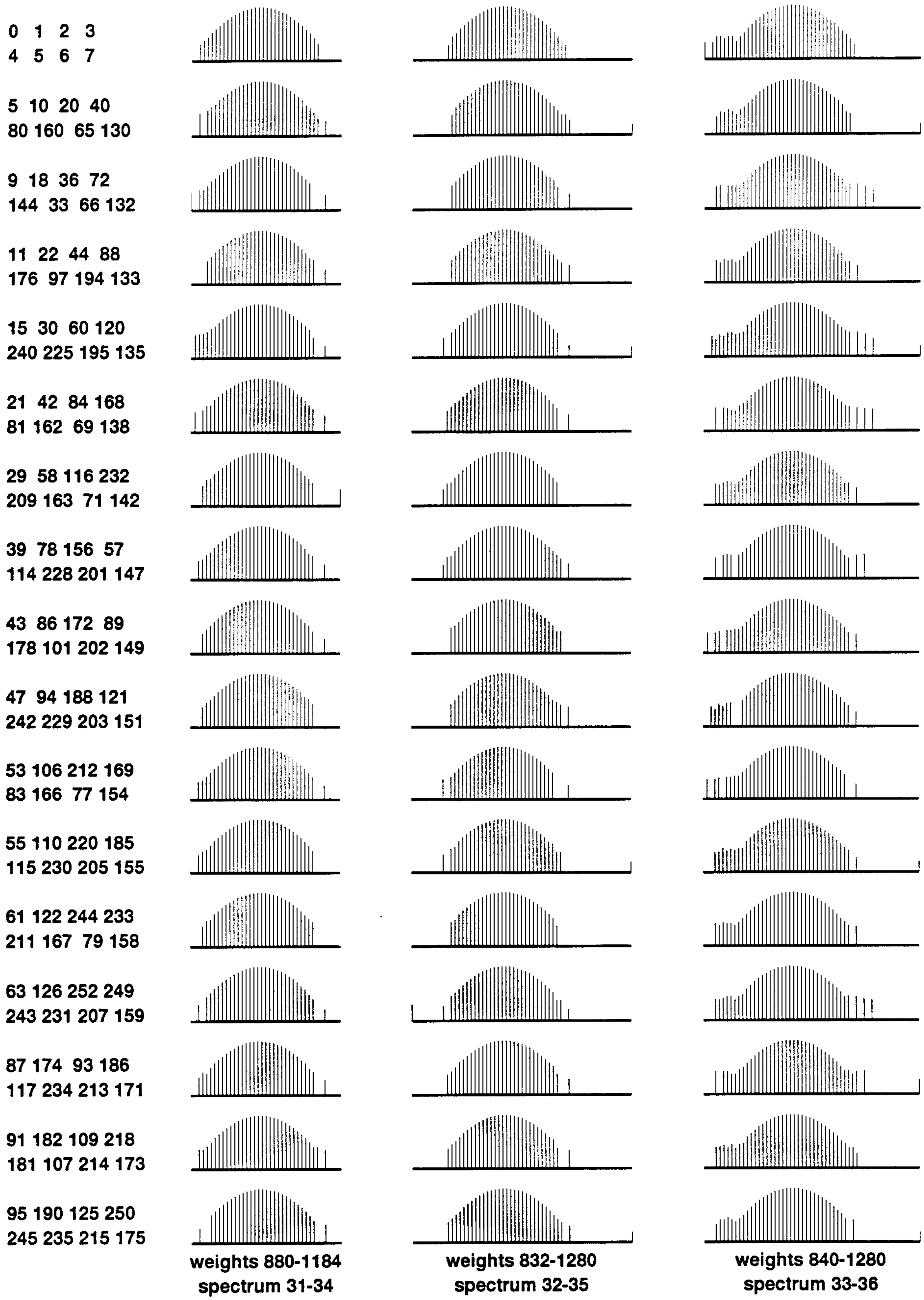
spectrum 30-33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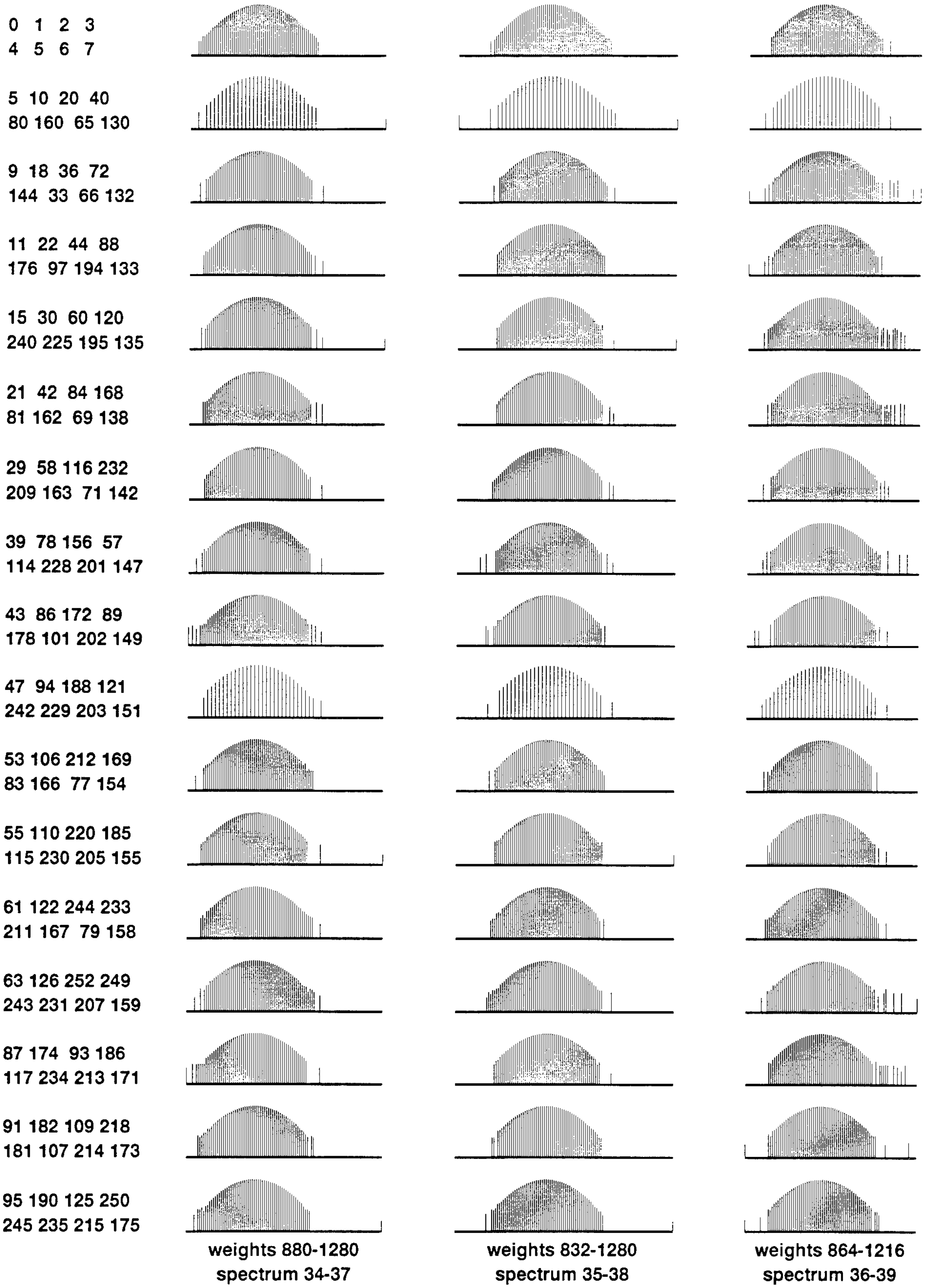
spectrum 32-35
weights 840-1280
spectrum 33-36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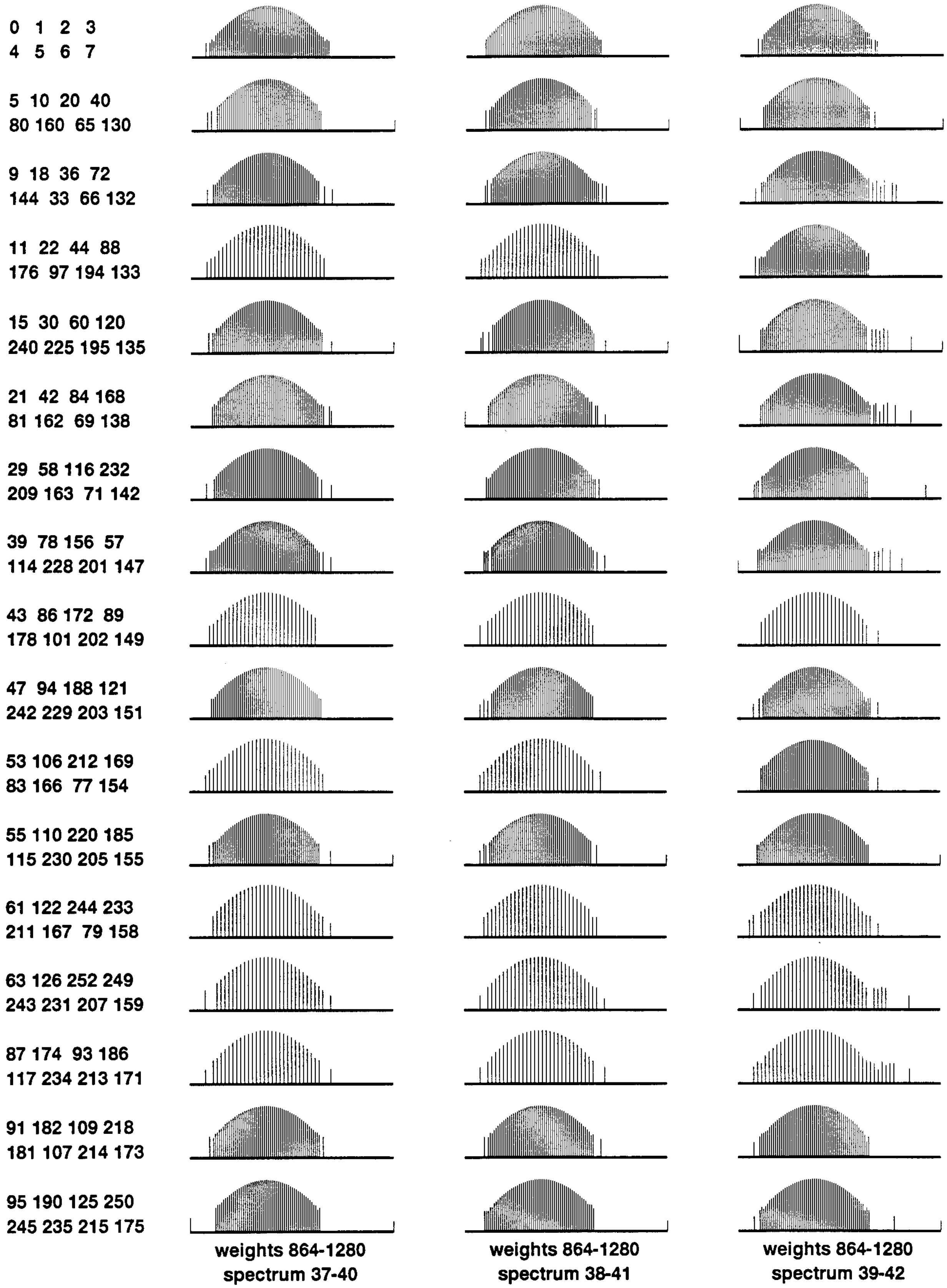
spectrum 35-38
weights 864-1216
spectrum 36-39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spectrum 38-41
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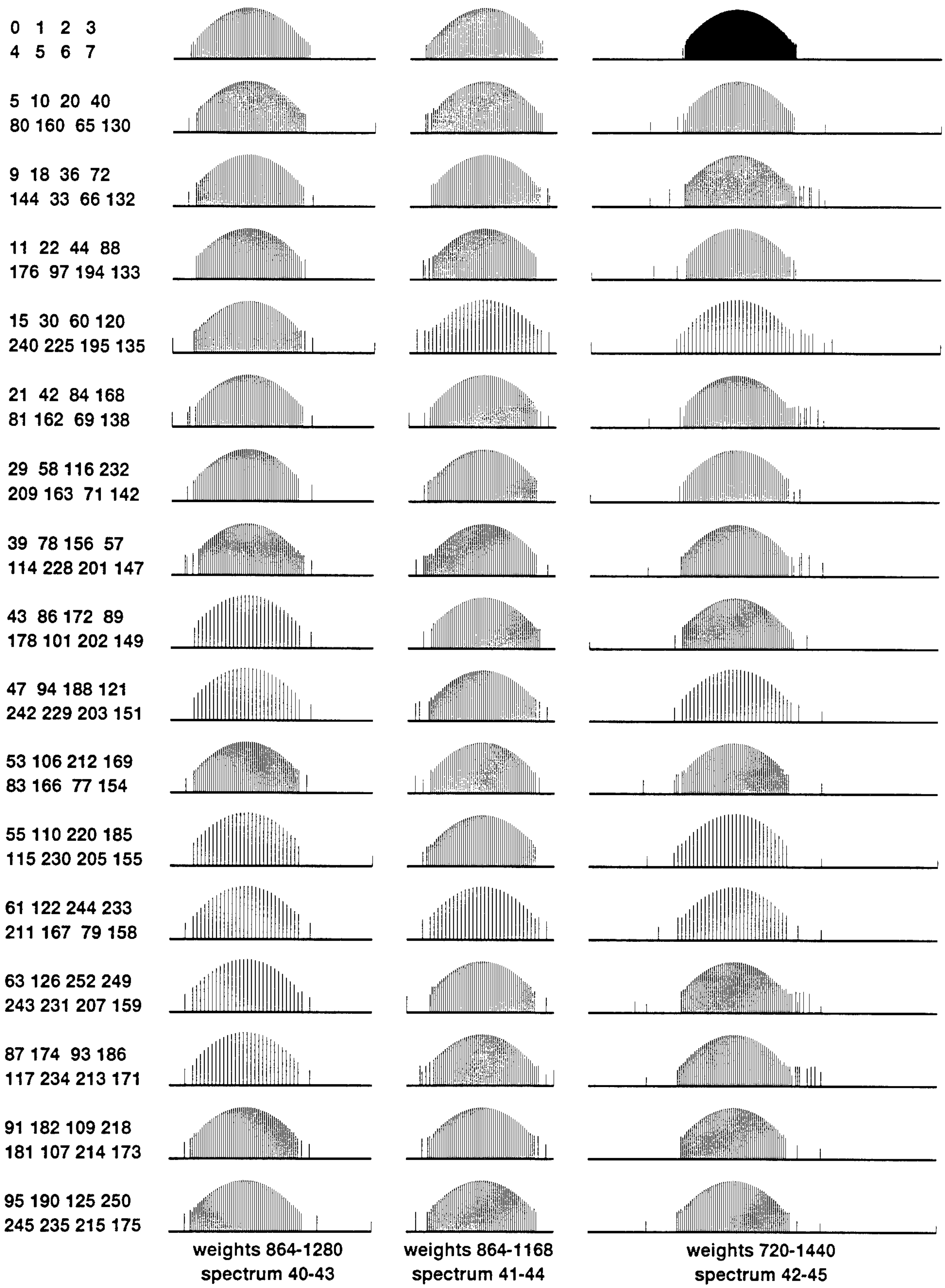
spectrum 39-42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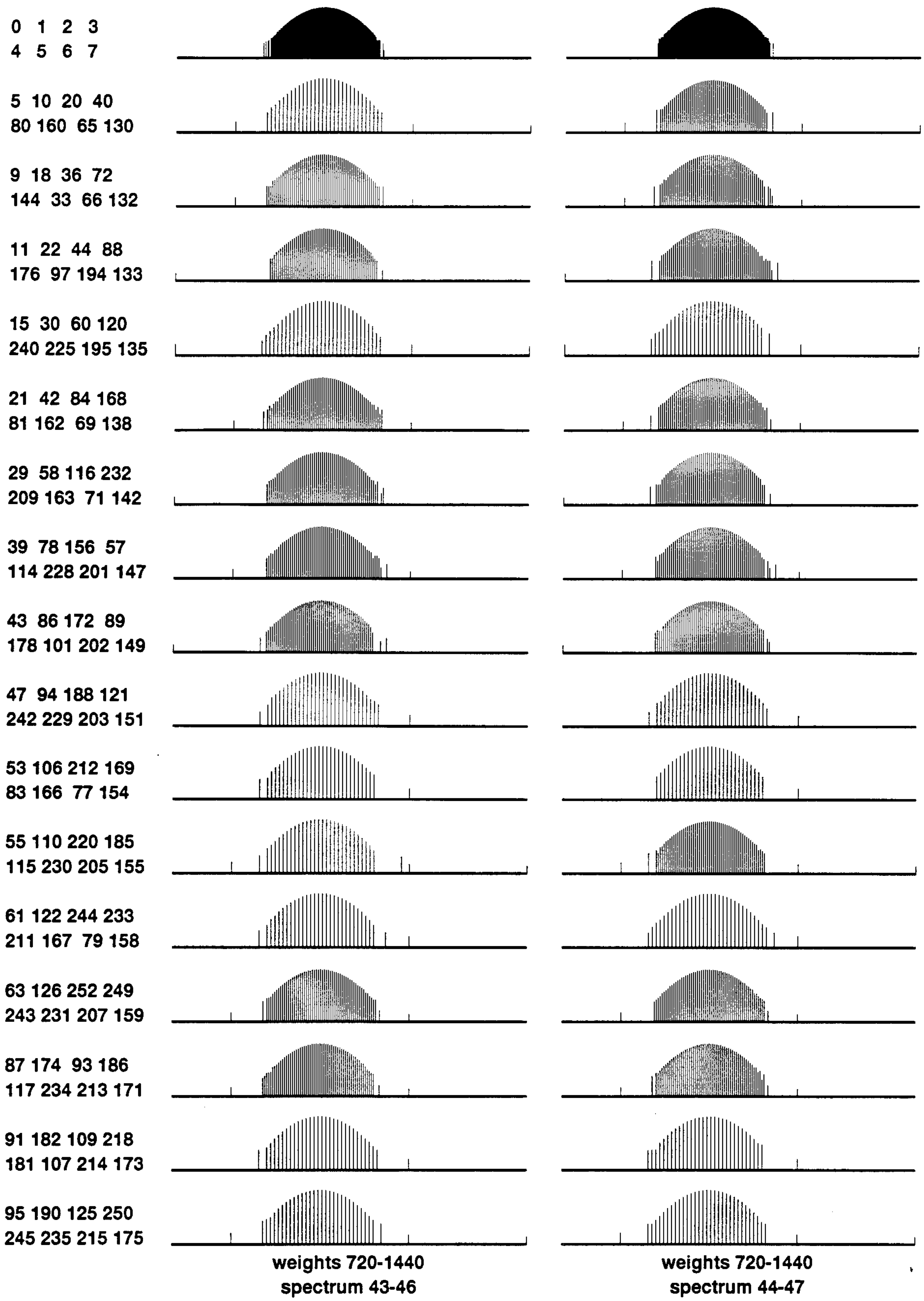
spectrum 41-44
weights 720-1440
spectrum 42-45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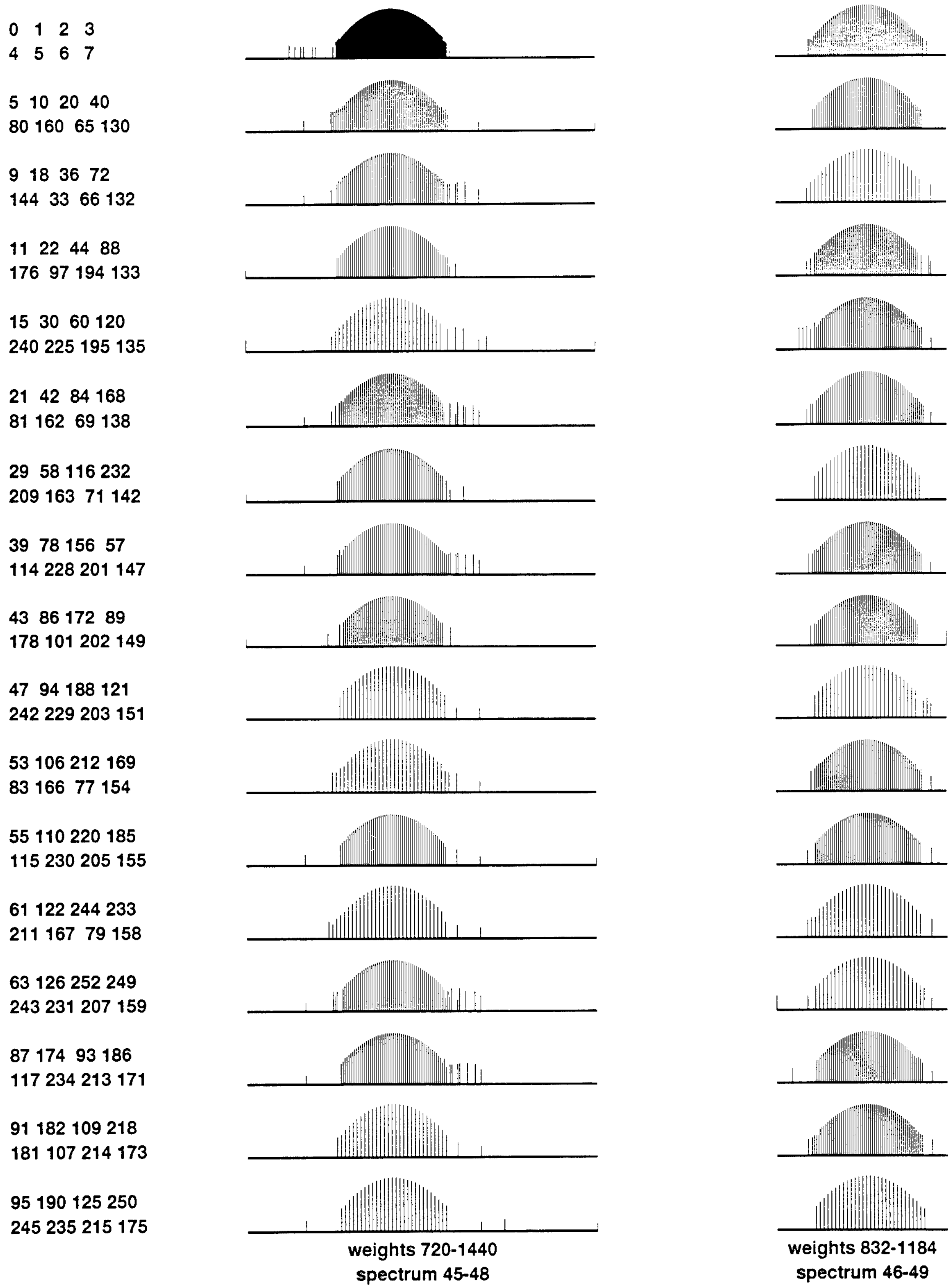
spectrum 44-47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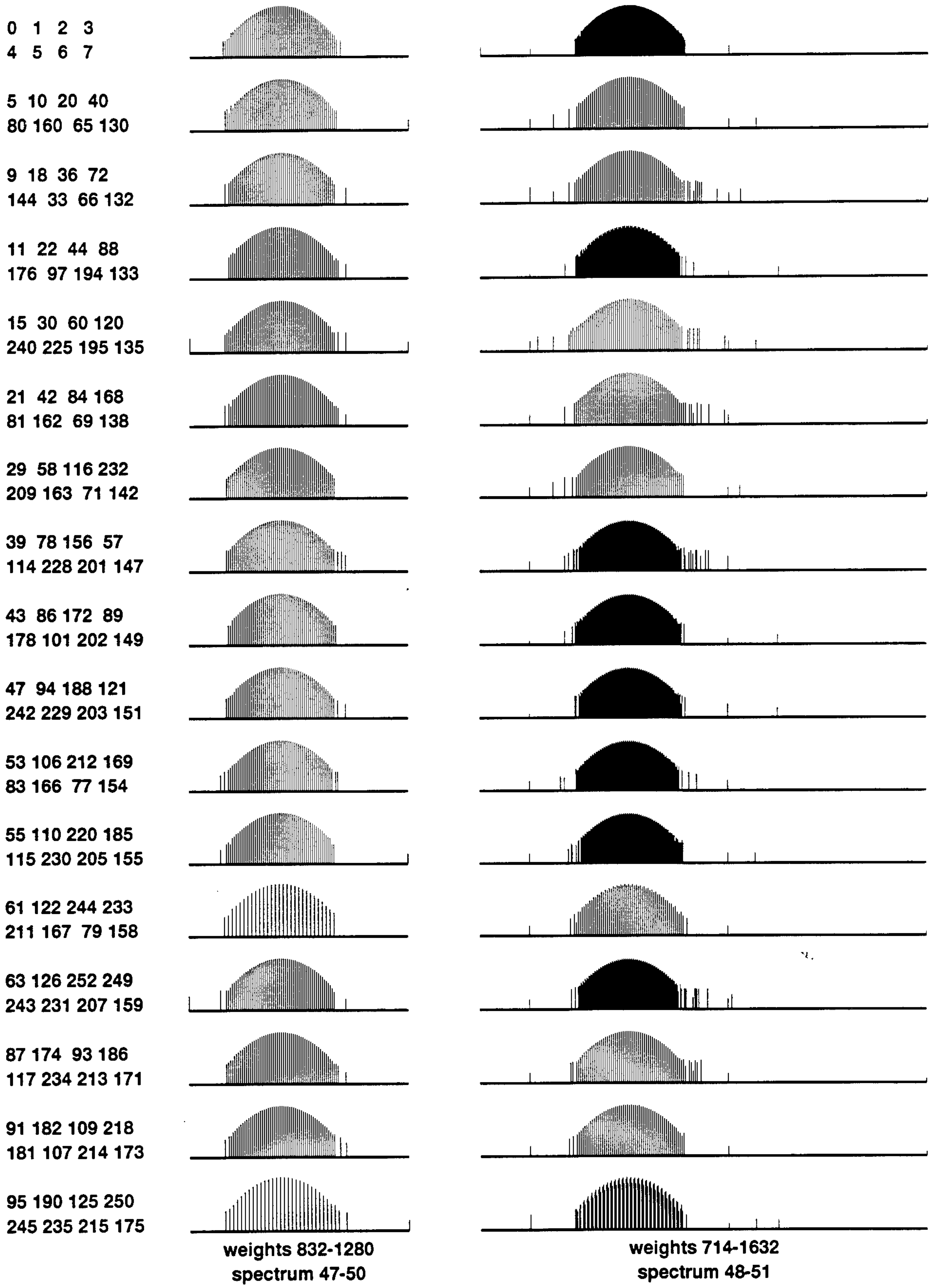
weights 832-1184
spectrum 46-49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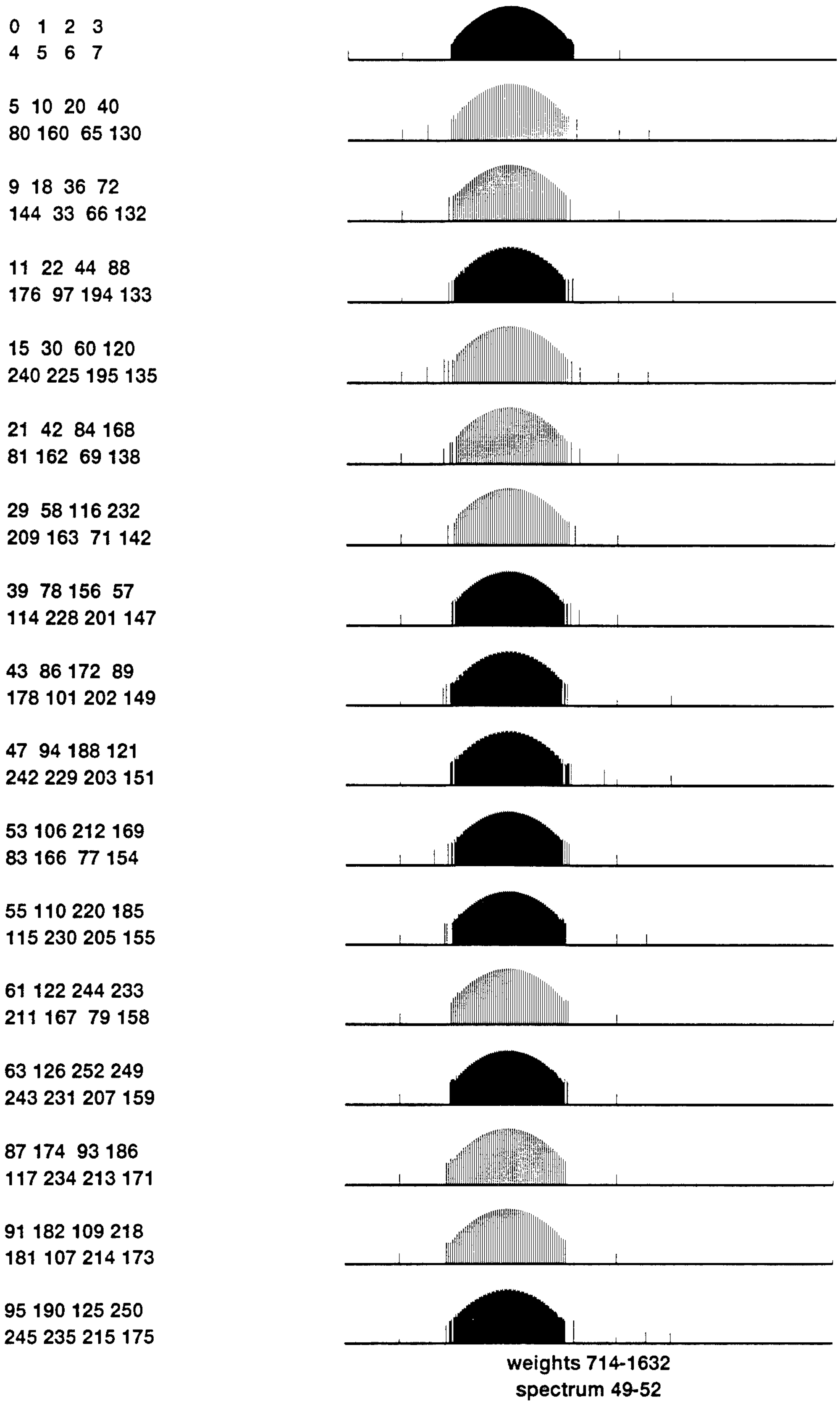
spectrum 47-50
weights 714-1632
spectrum 48-51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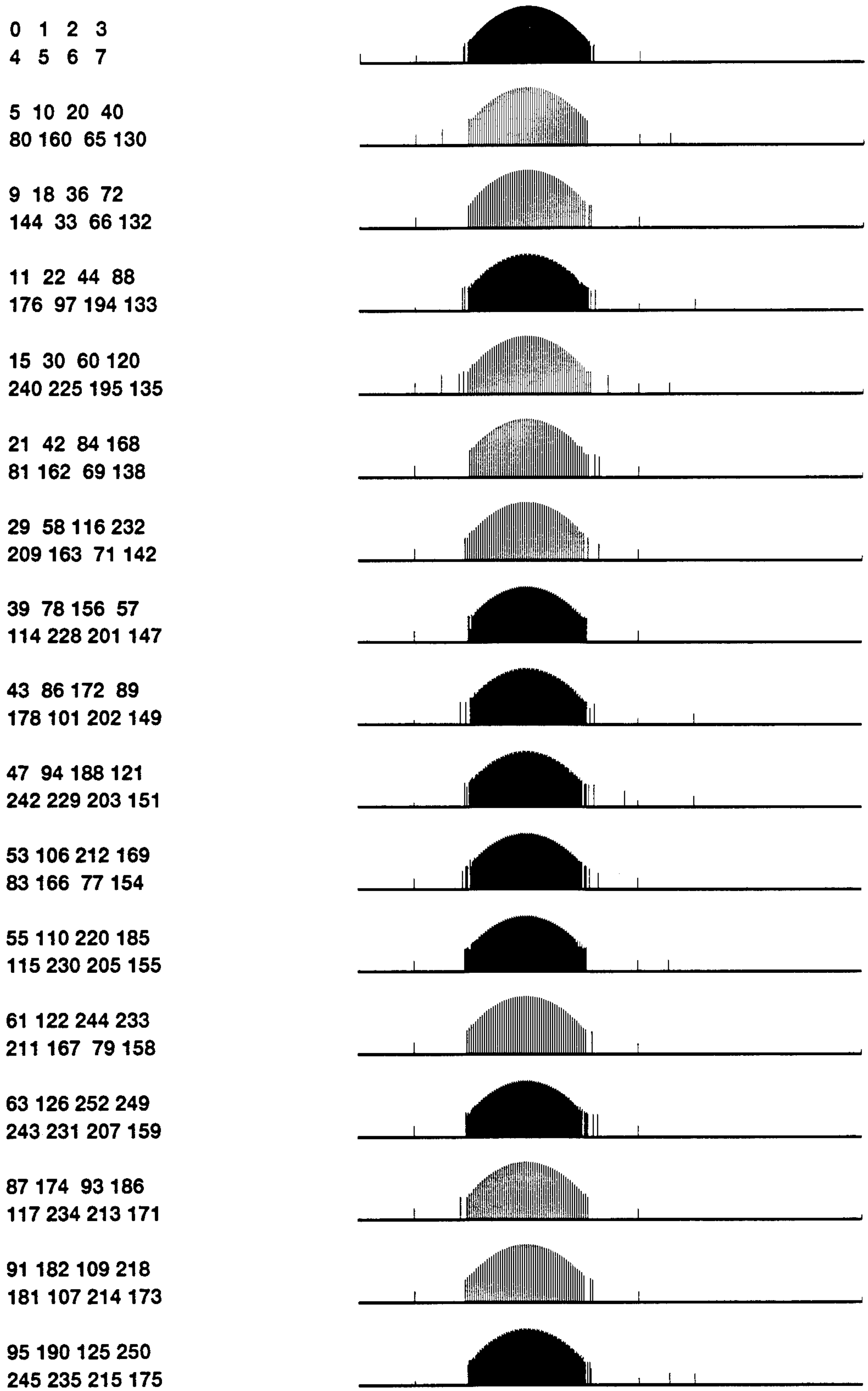
spectrum 49-52



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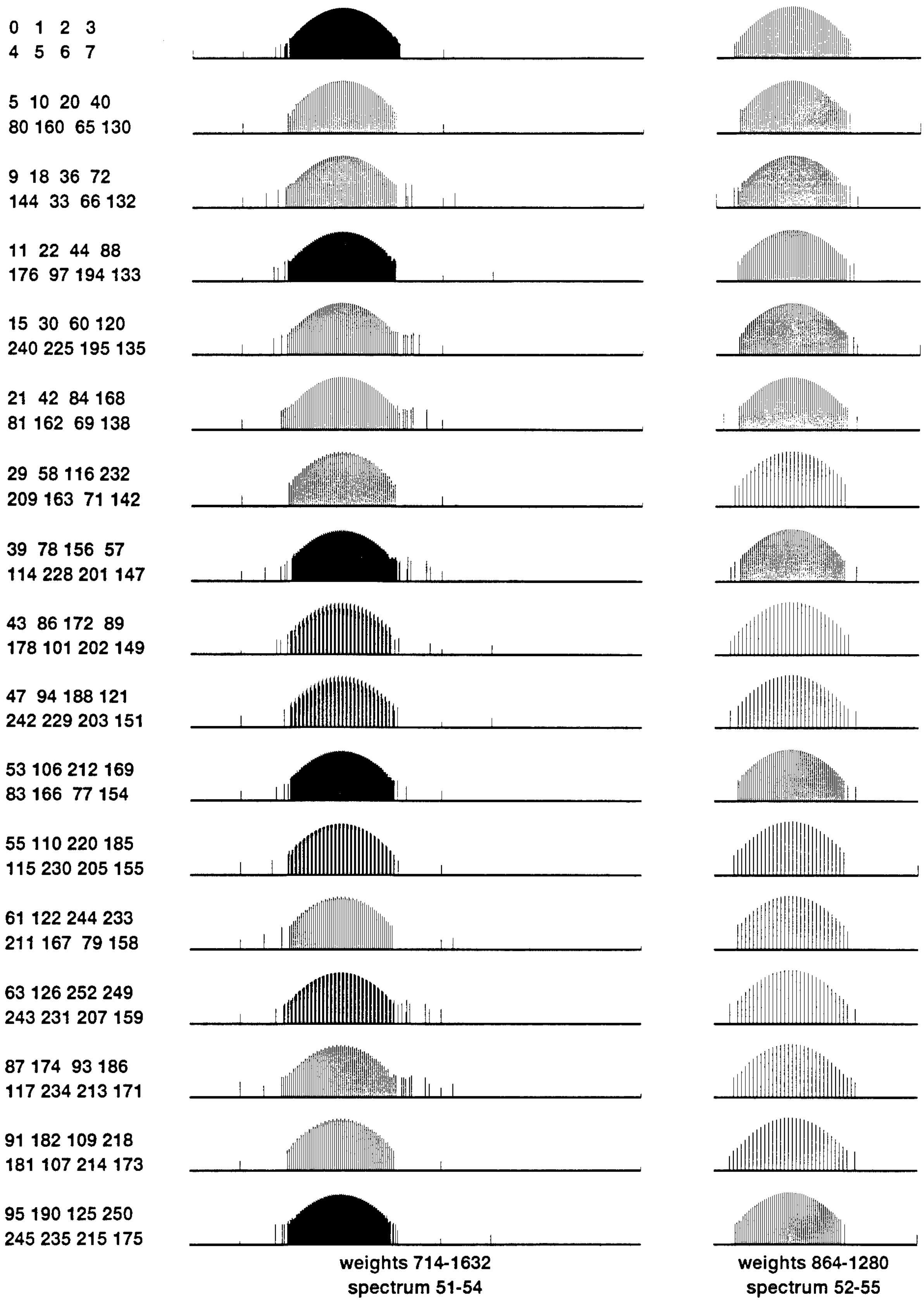
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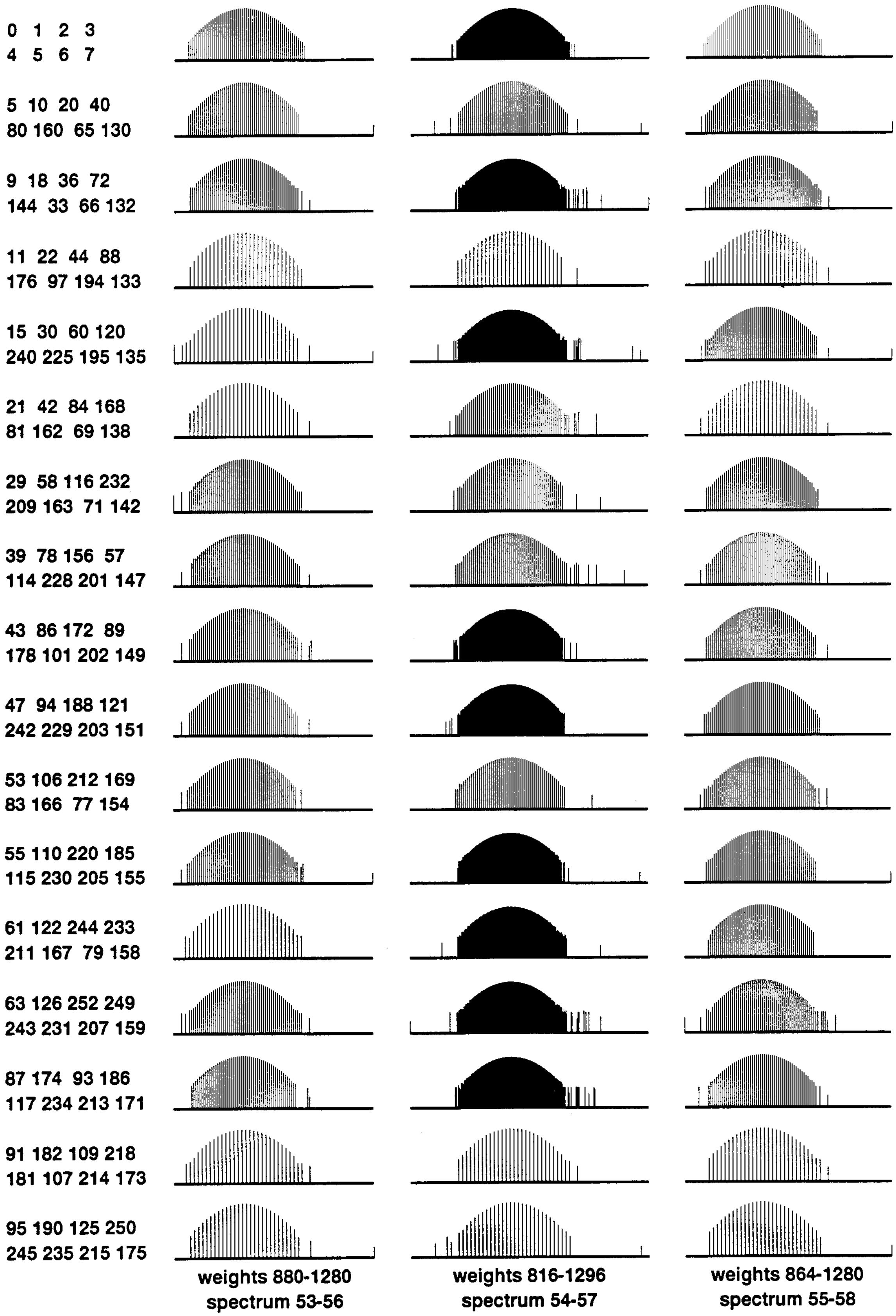
weights 864-1280
spectrum 52-55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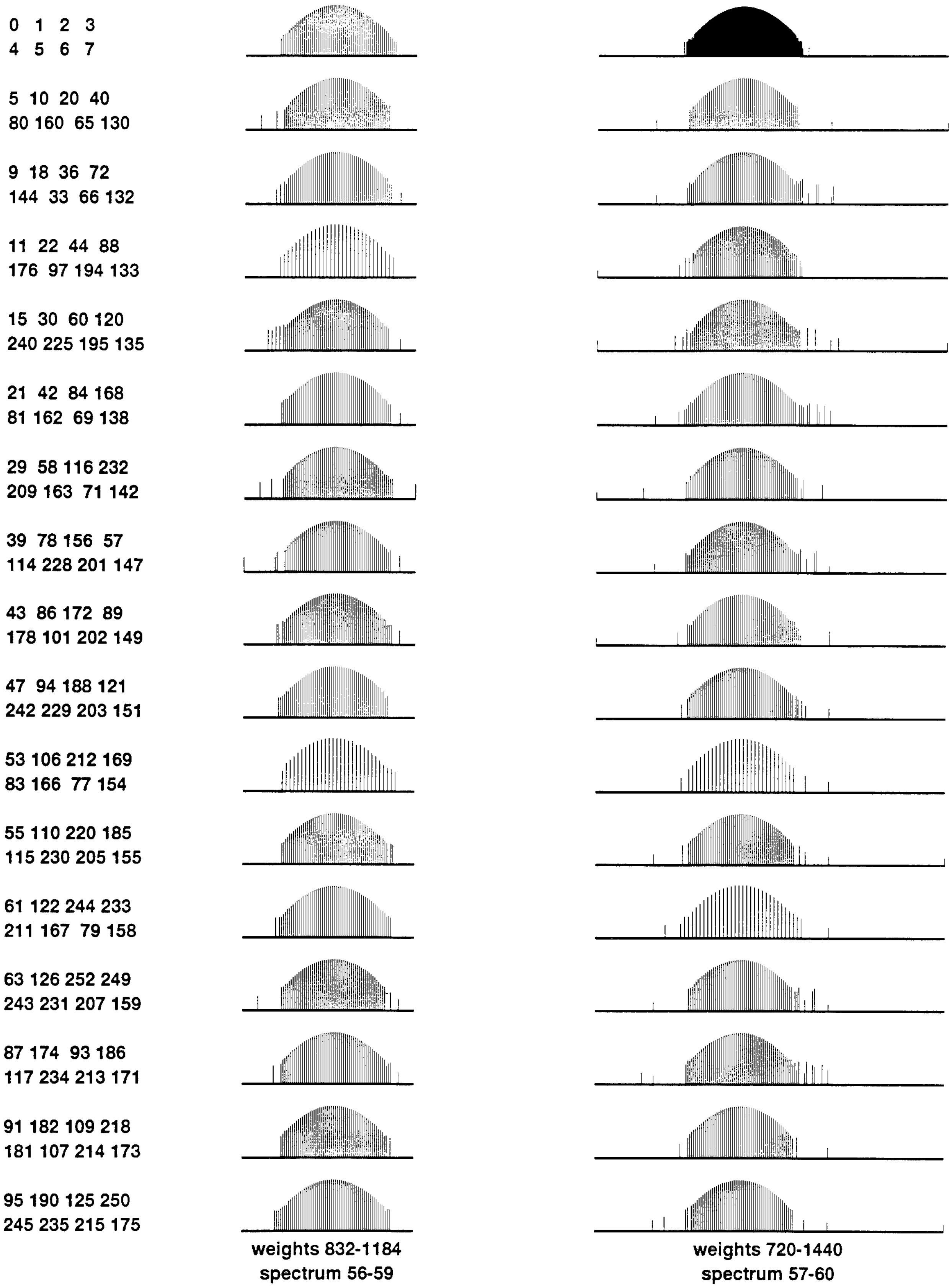
weights 816-1296
spectrum 54-57
weights 864-1280
spectrum 55-58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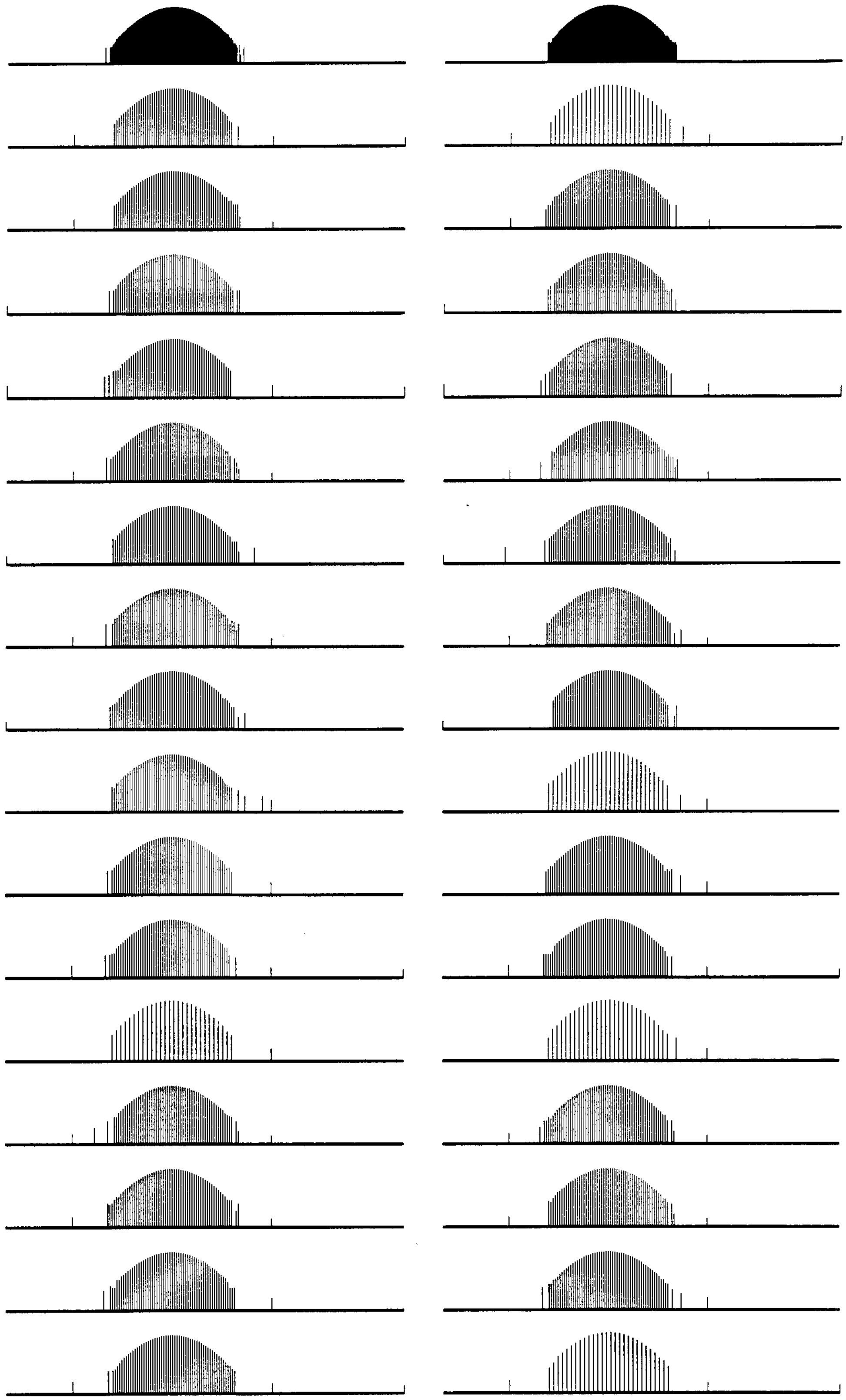
weights 720-1440
spectrum 57-60



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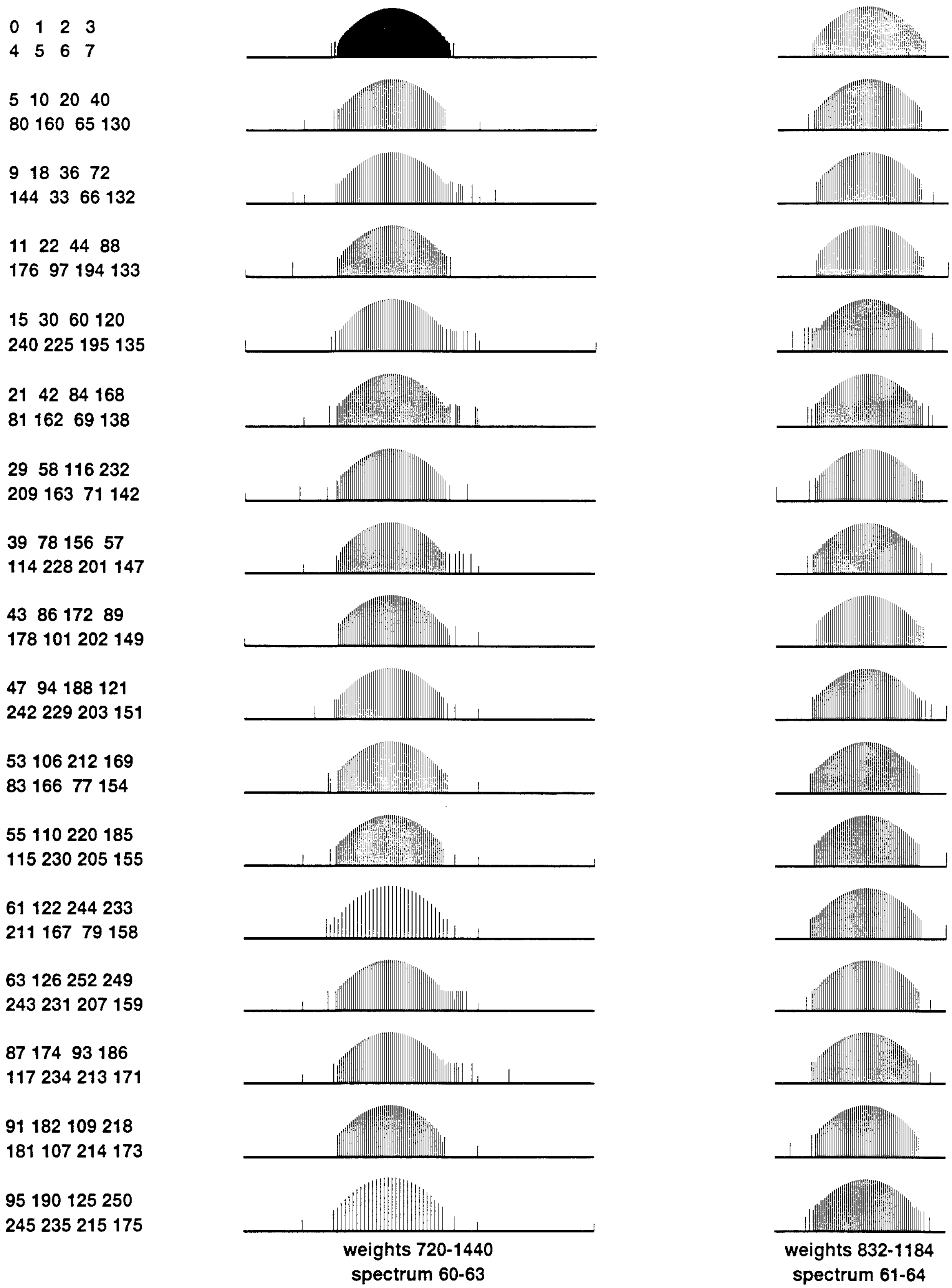
spectrum 58-61

weights 720-1440
spectrum 59-62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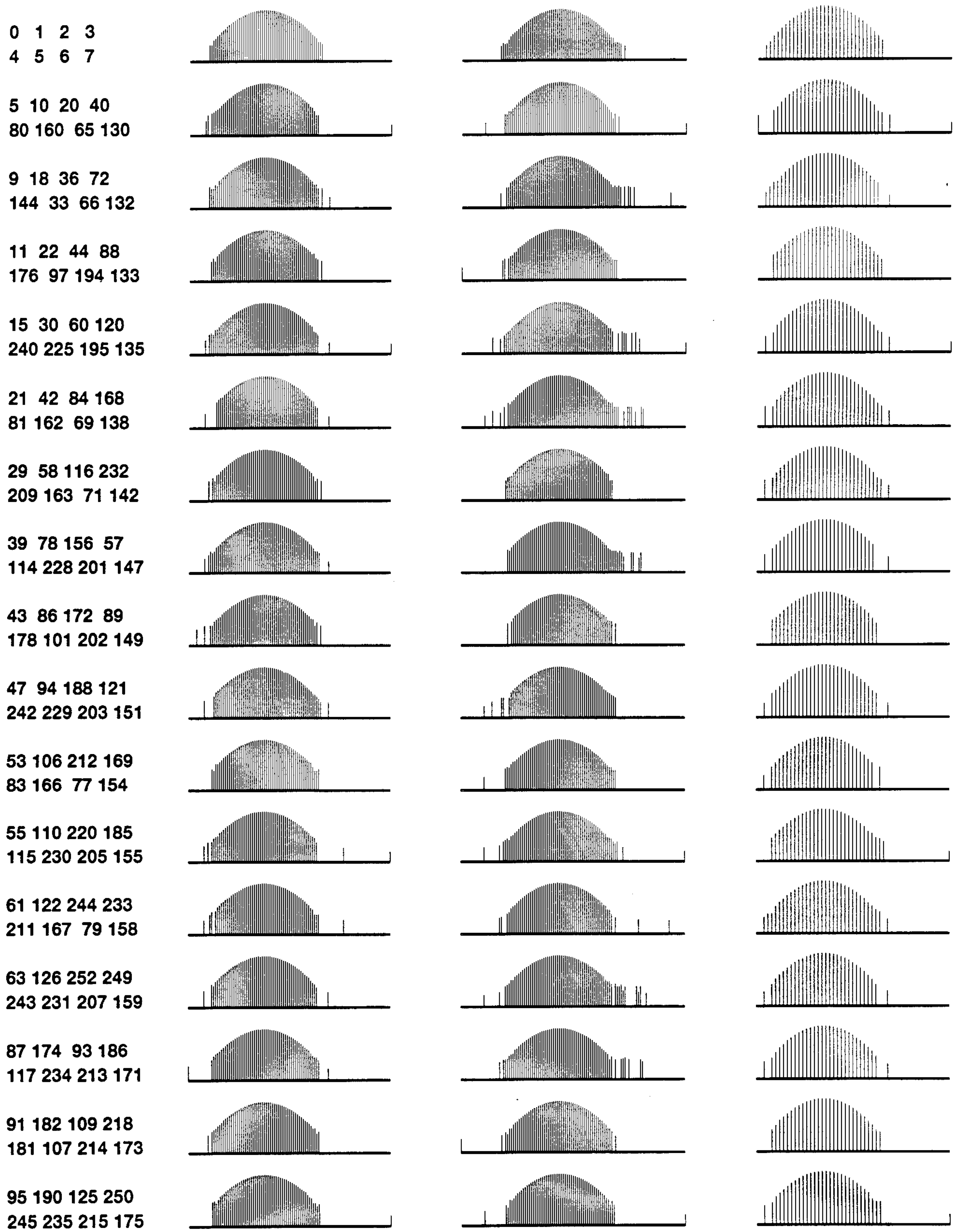
spectrum 61-64



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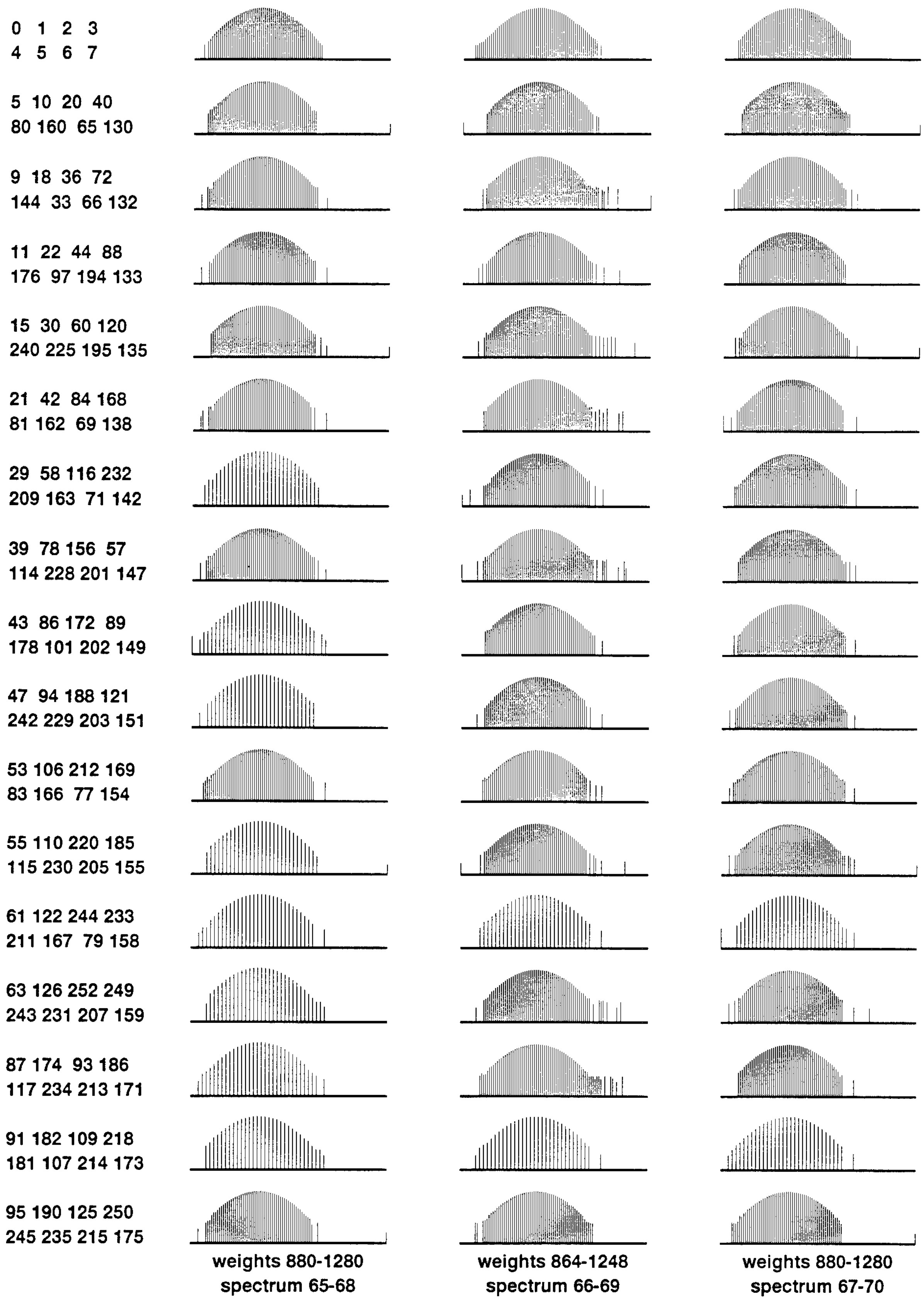
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spectrum 66-69
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spectrum 67-70

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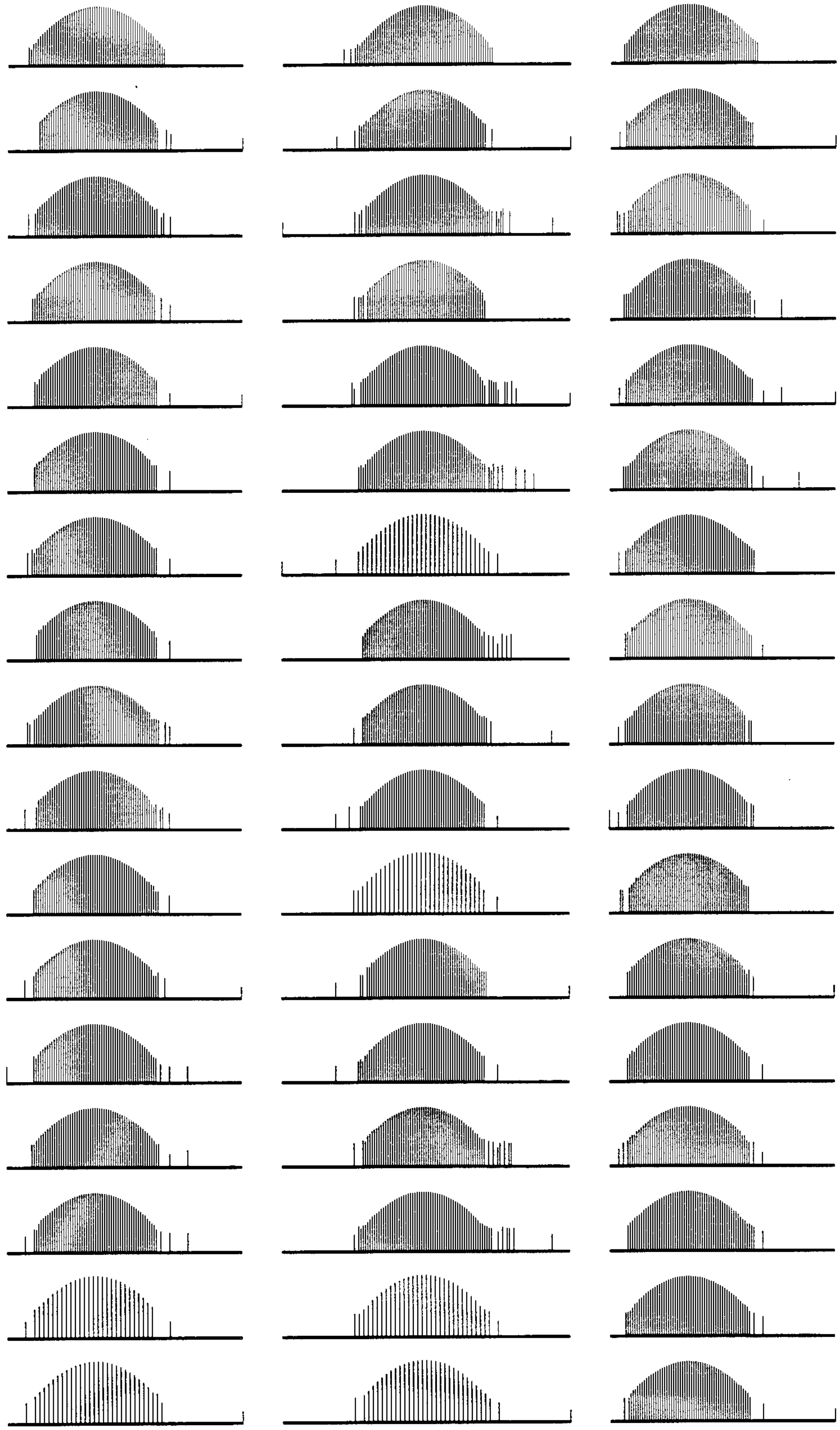

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spectrum 69-72

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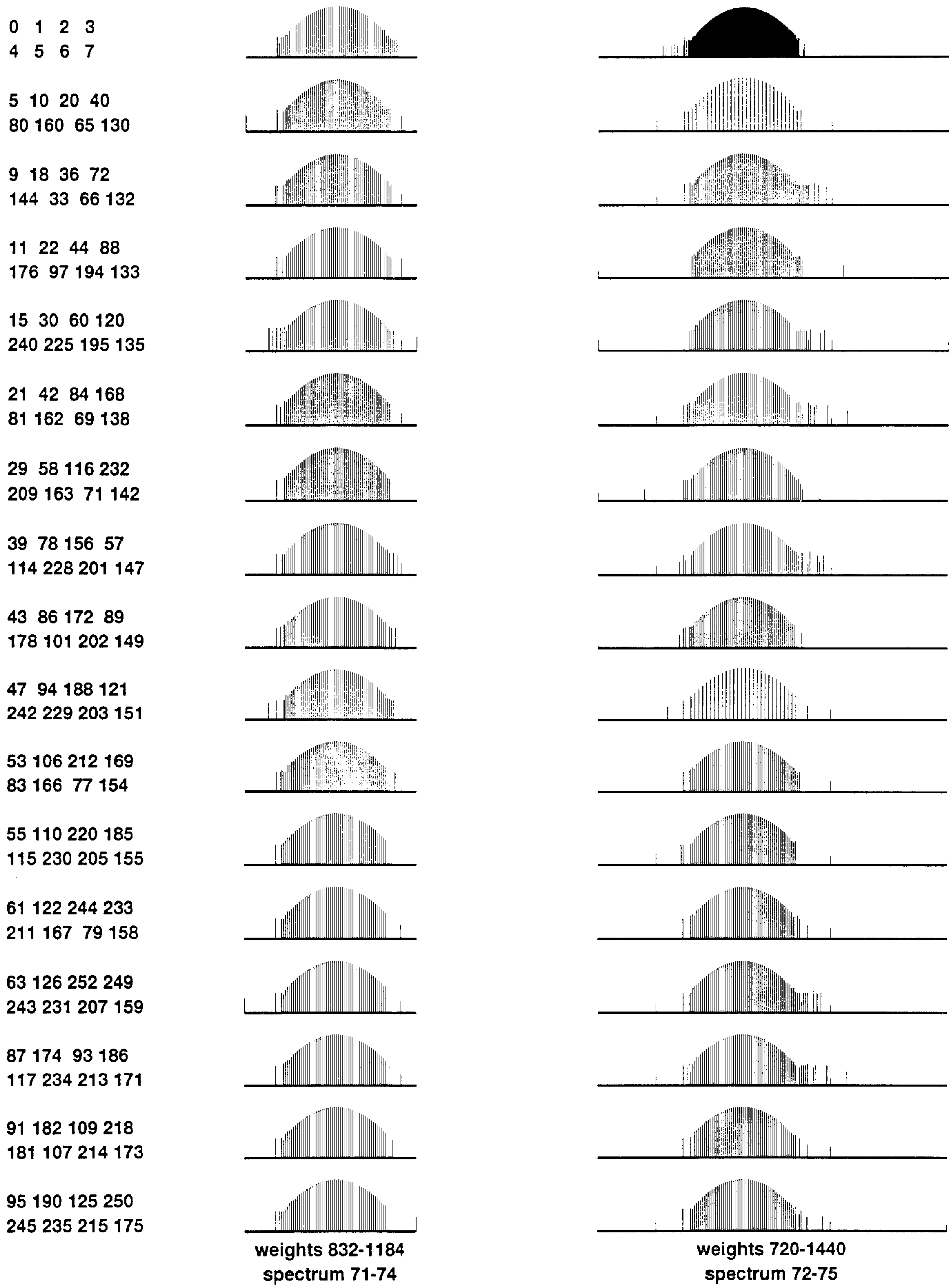
spectrum 70-73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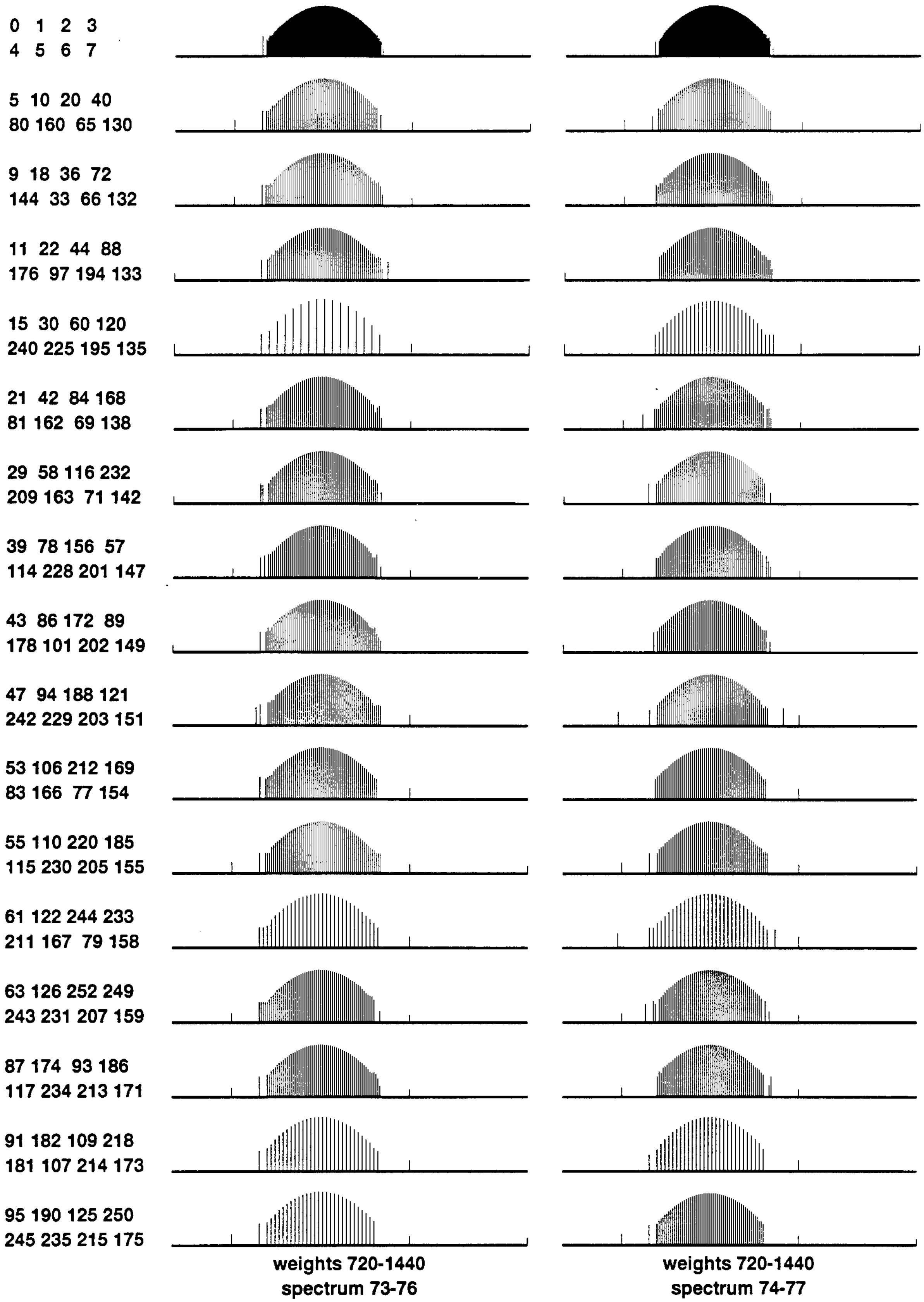
weights 720-1440
spectrum 72-75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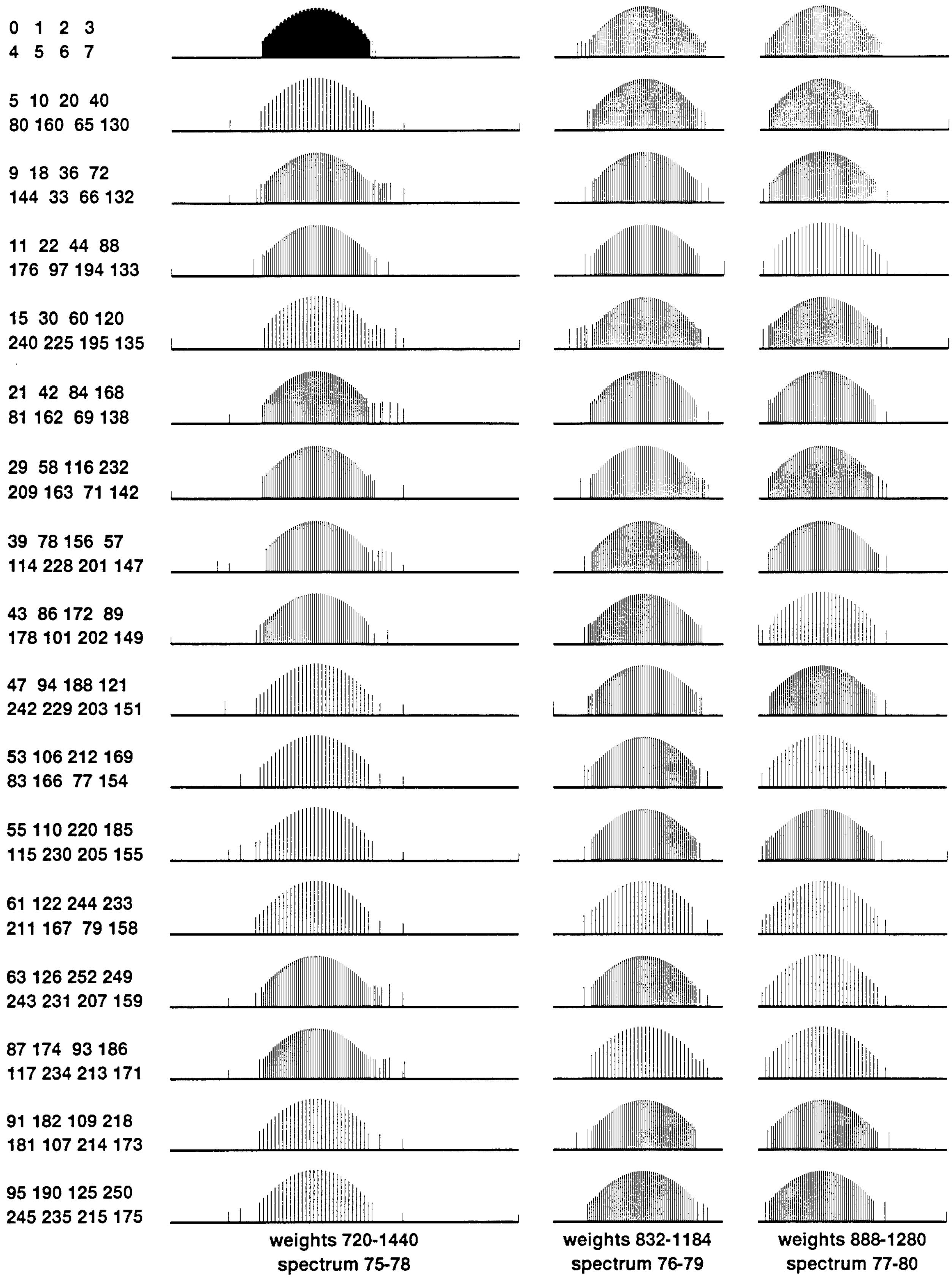
weights 720-1440
spectrum 74-77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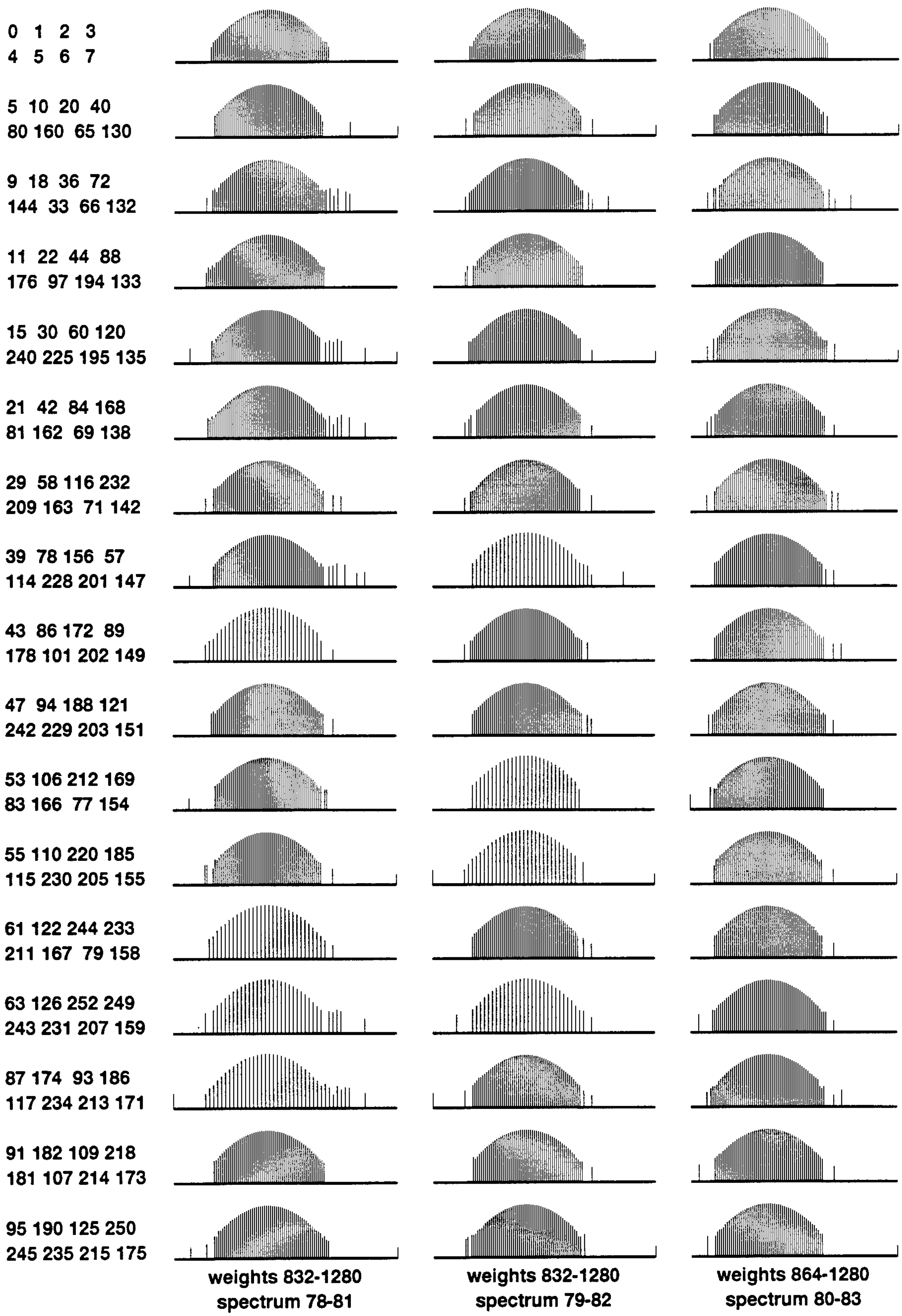
spectrum 77-80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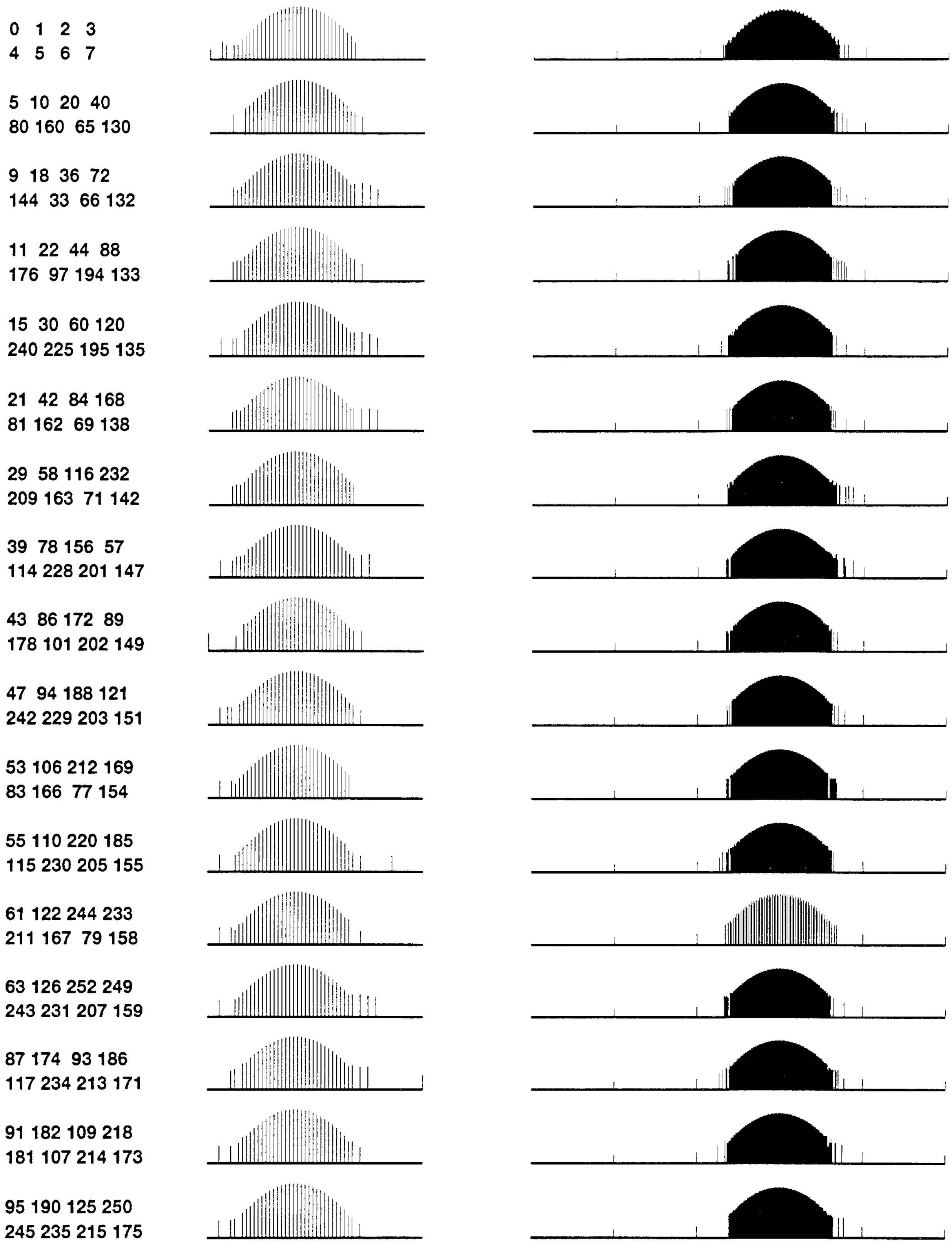
weights 832-1280
spectrum 79-82
weights 864-1280
spectrum 80-83



weights 840-1280
spectrum 81-84

weights 510-1360
spectrum 82-85

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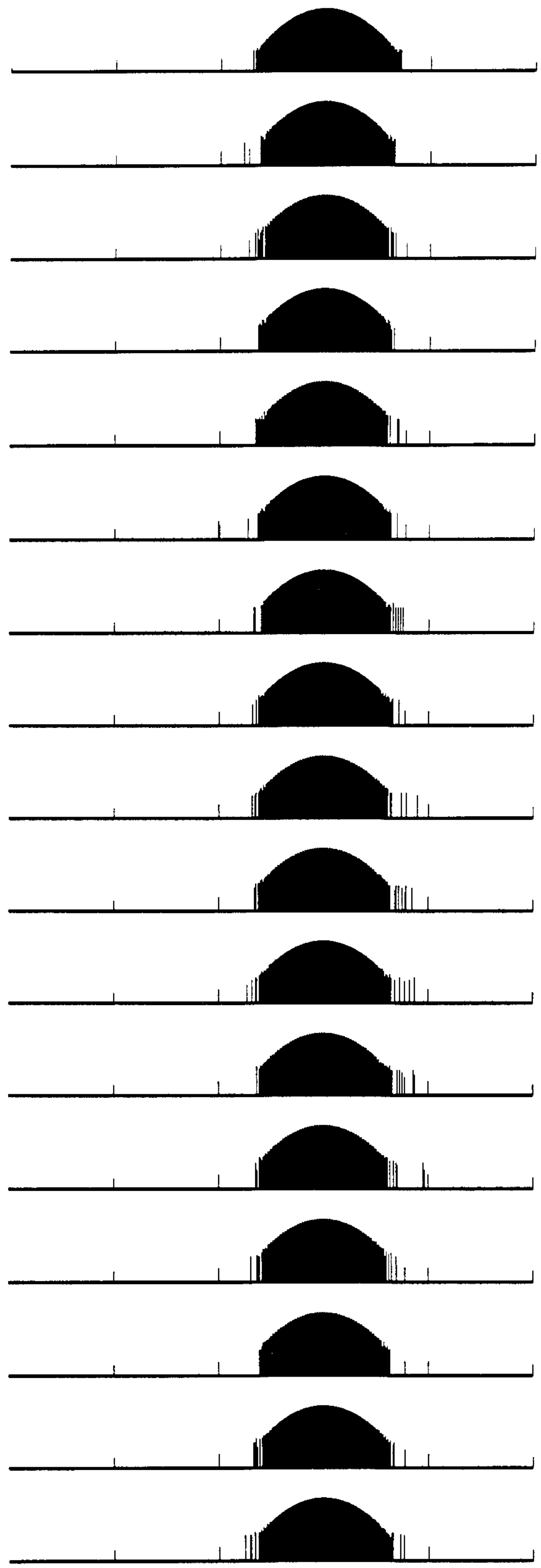
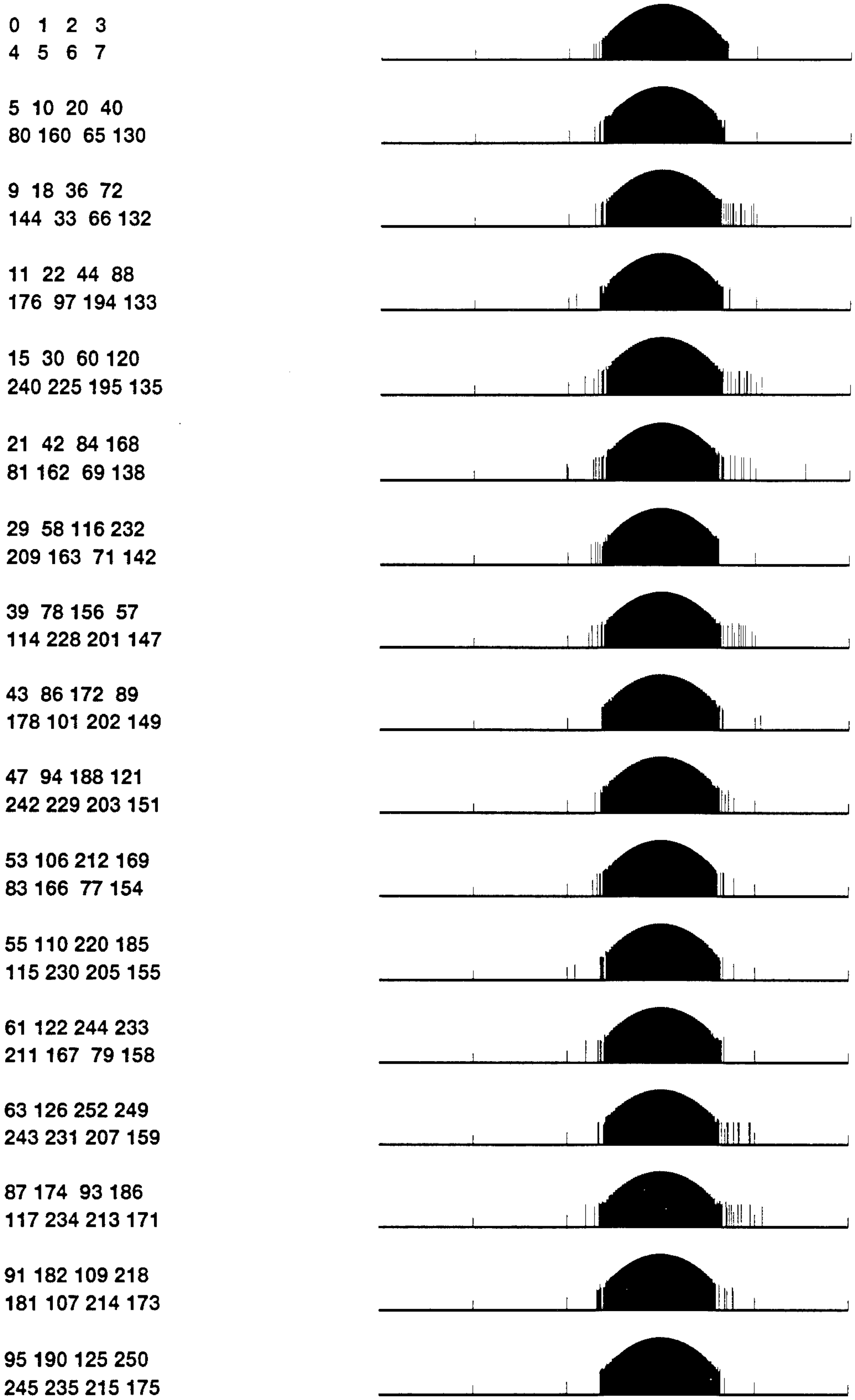
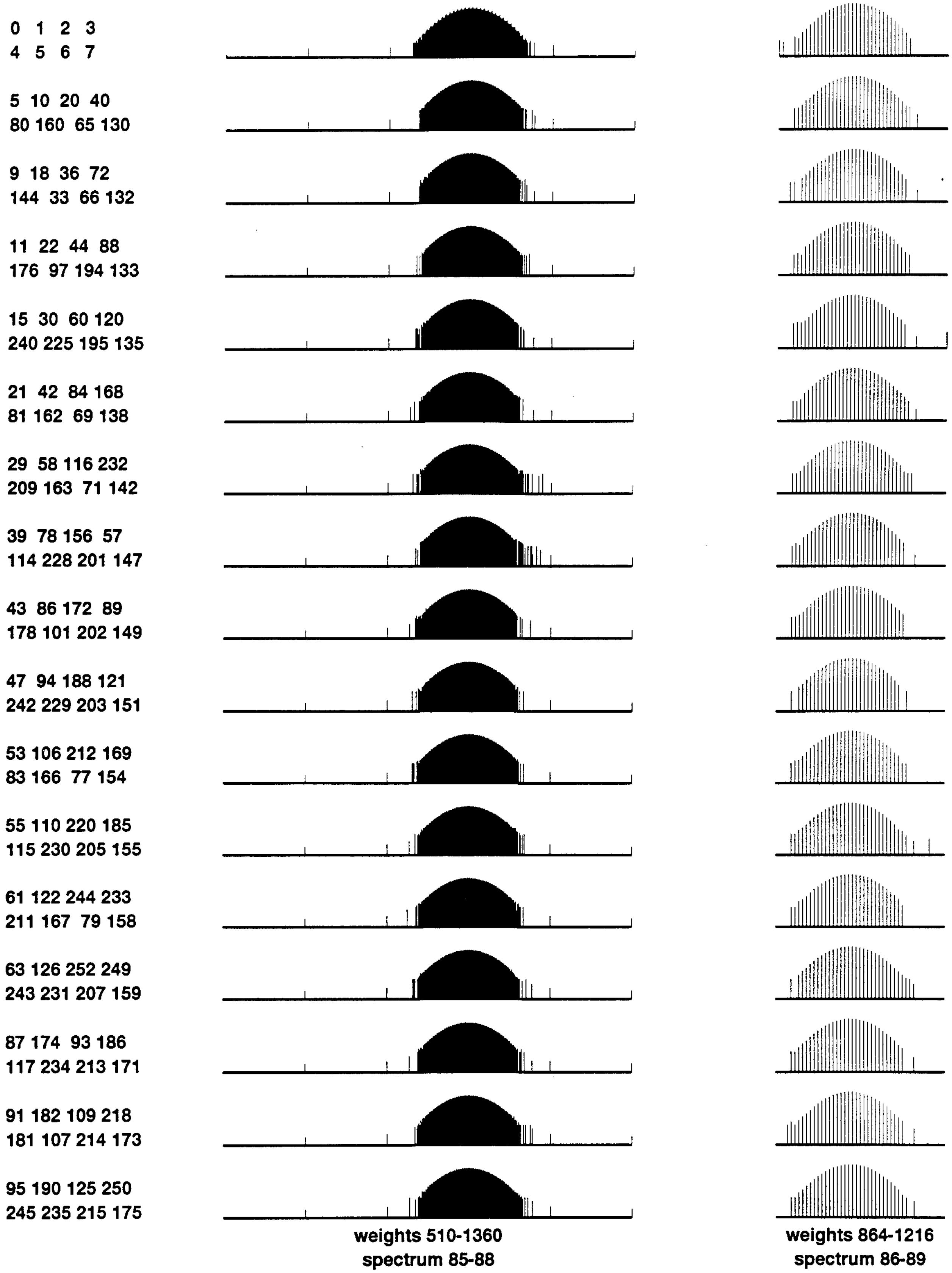


weights 510-1360
spectrum 83-86



weights 510-1360
spectrum 84-87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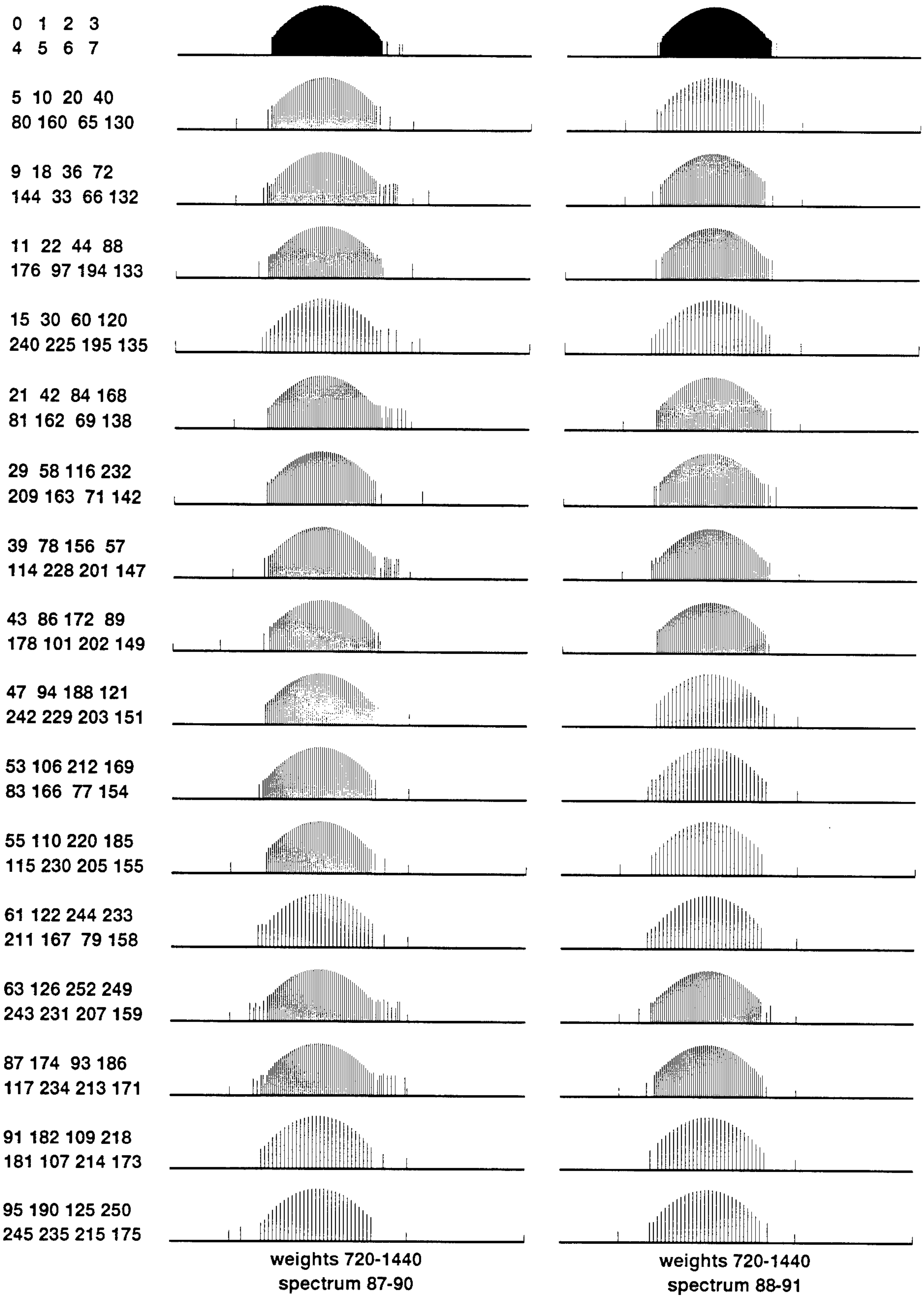
weights 510-1360
spectrum 85-88
weights 864-1216
spectrum 86-89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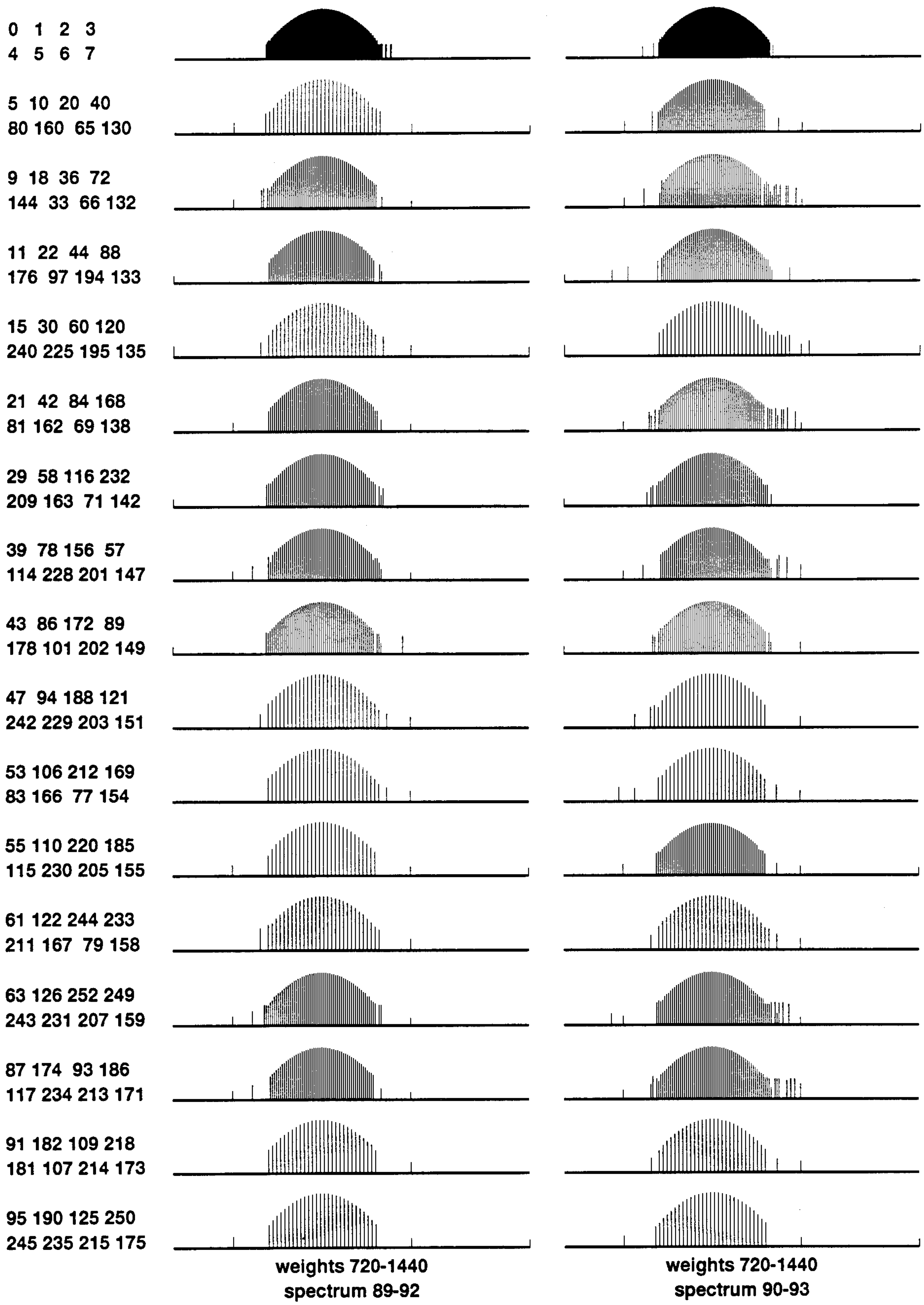
weights 720-1440
spectrum 87-90
weights 720-1440
spectrum 88-91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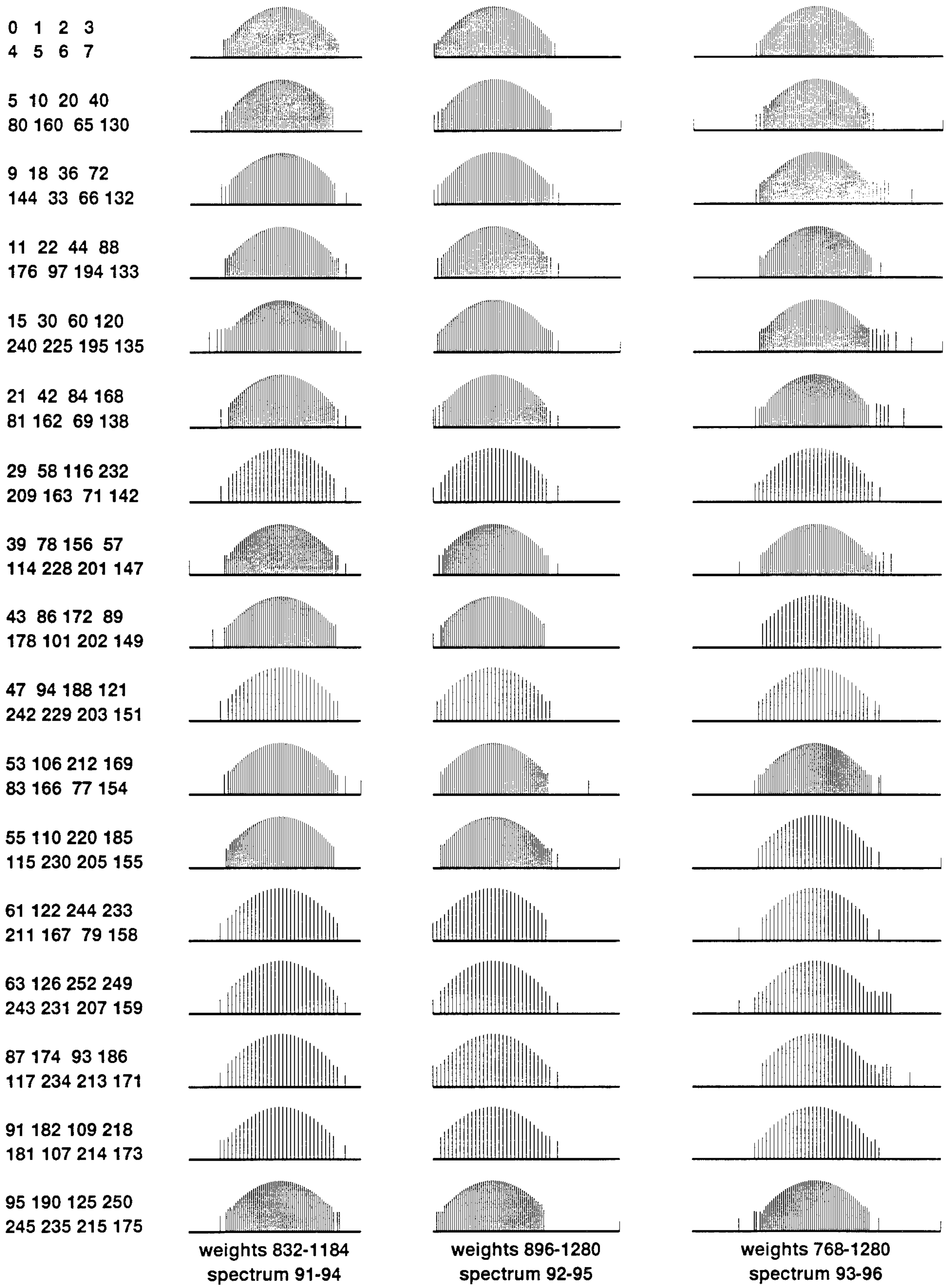
weights 720-1440
spectrum 89-92
weights 720-1440
spectrum 90-93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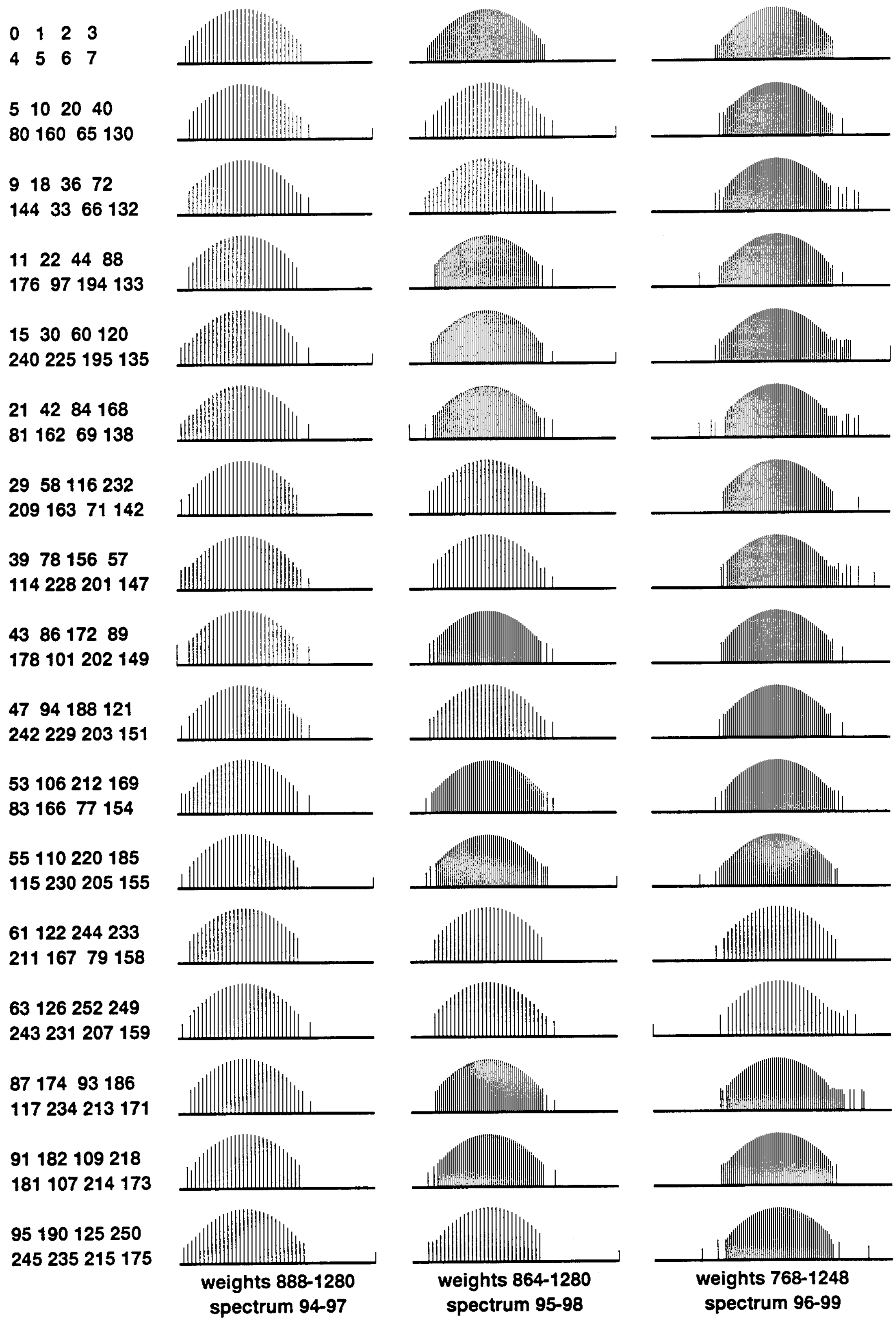
weights 832-1184
spectrum 91-94
weights 896-1280
spectrum 92-95
weights 768-1280
spectrum 93-96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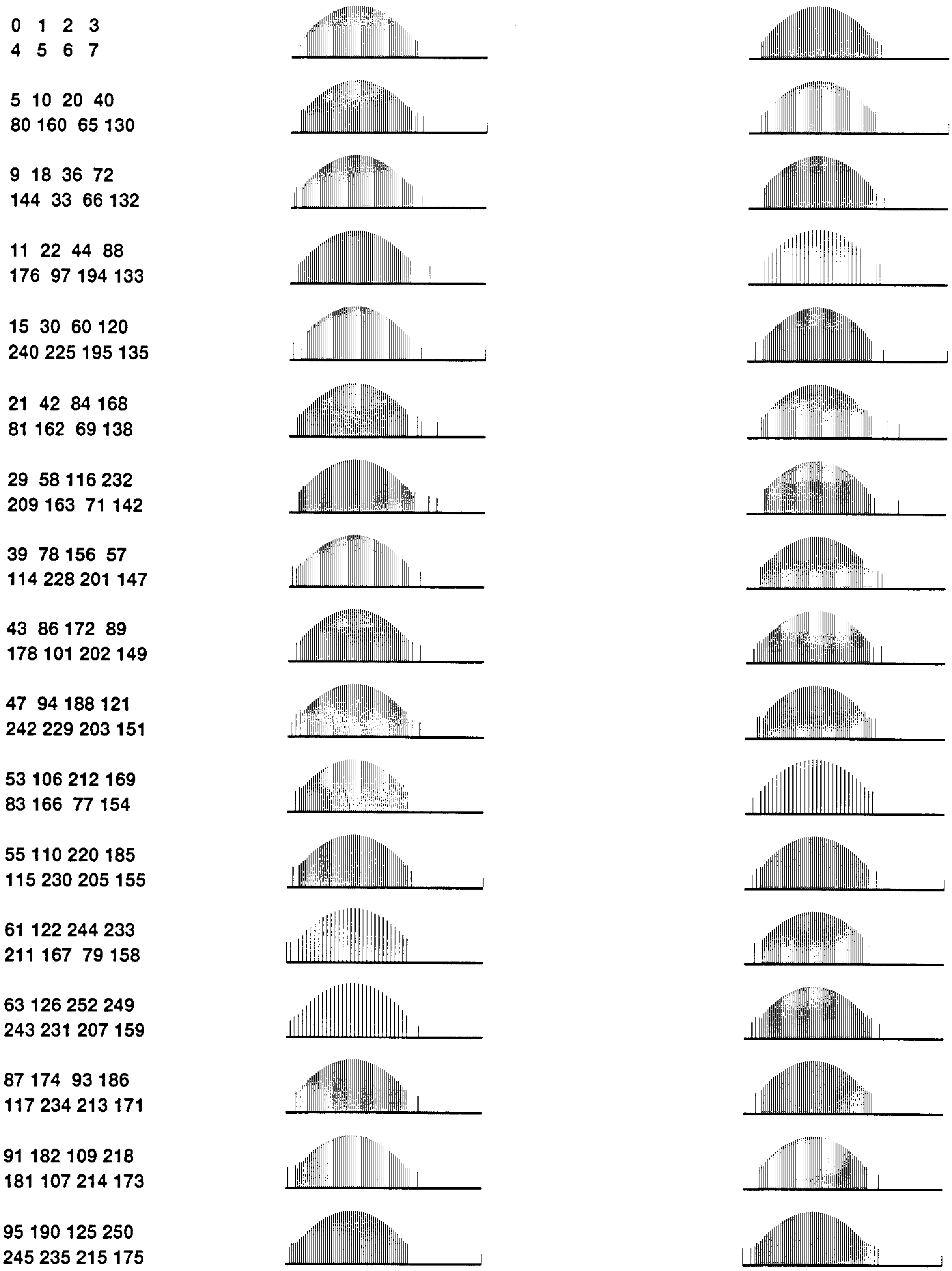
weights 888-1280
spectrum 94-97
weights 864-1280
spectrum 95-98
weights 768-1248
spectrum 96-99



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spectrum 97-100

weights 880-1280
spectrum 98-101

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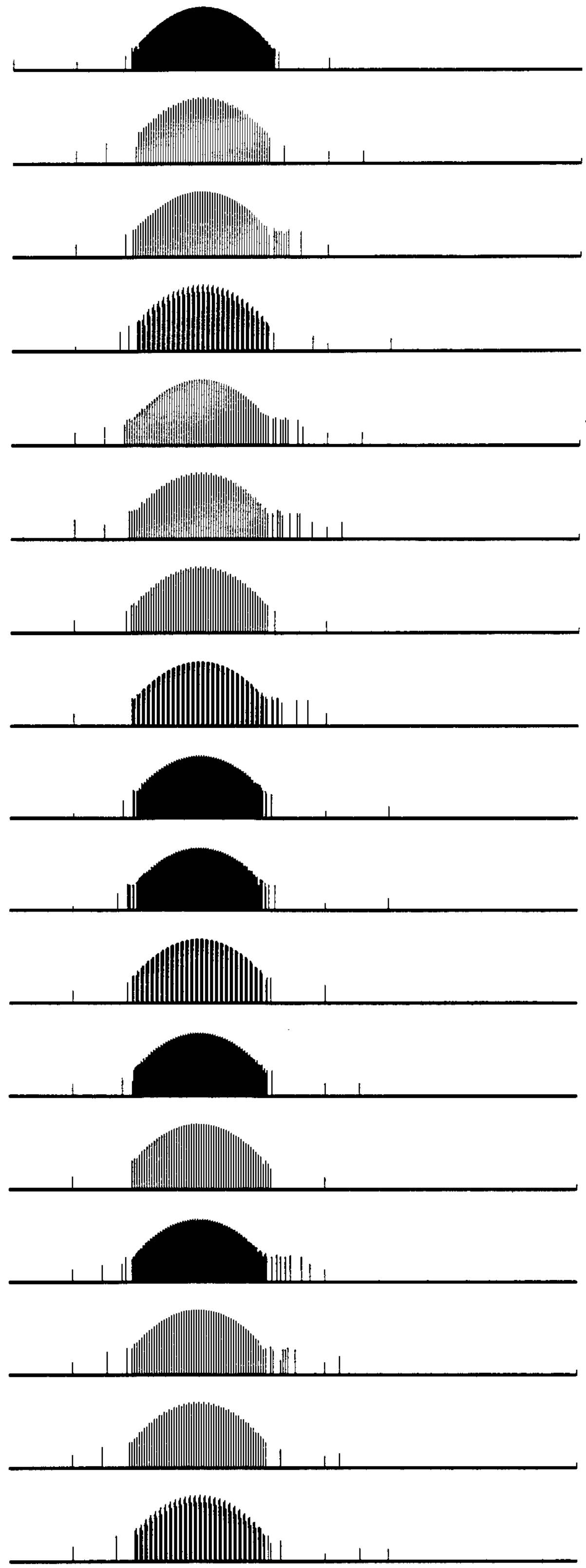
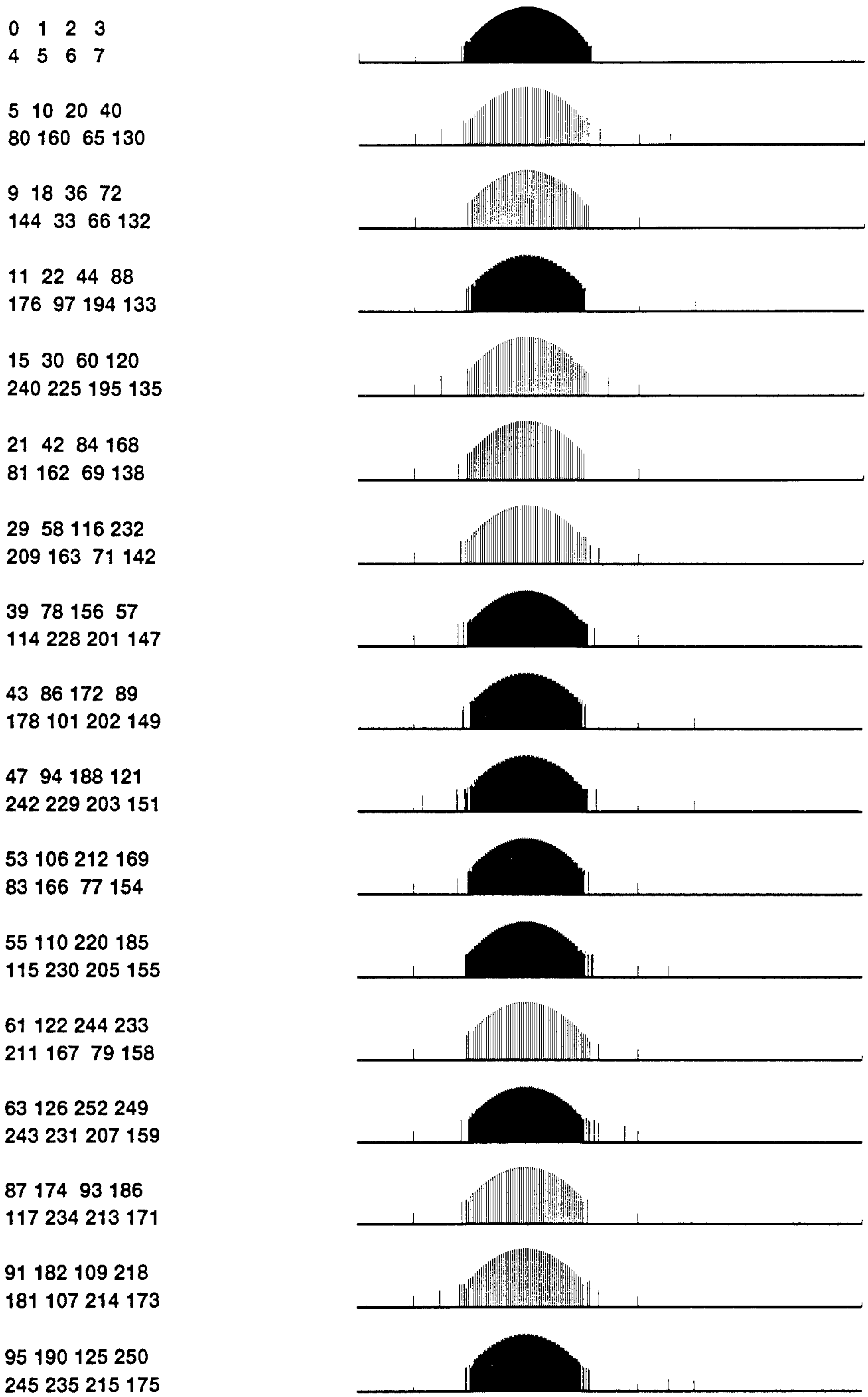


weights 714-1632
spectrum 99-102



weights 714-1632
spectrum 100-103

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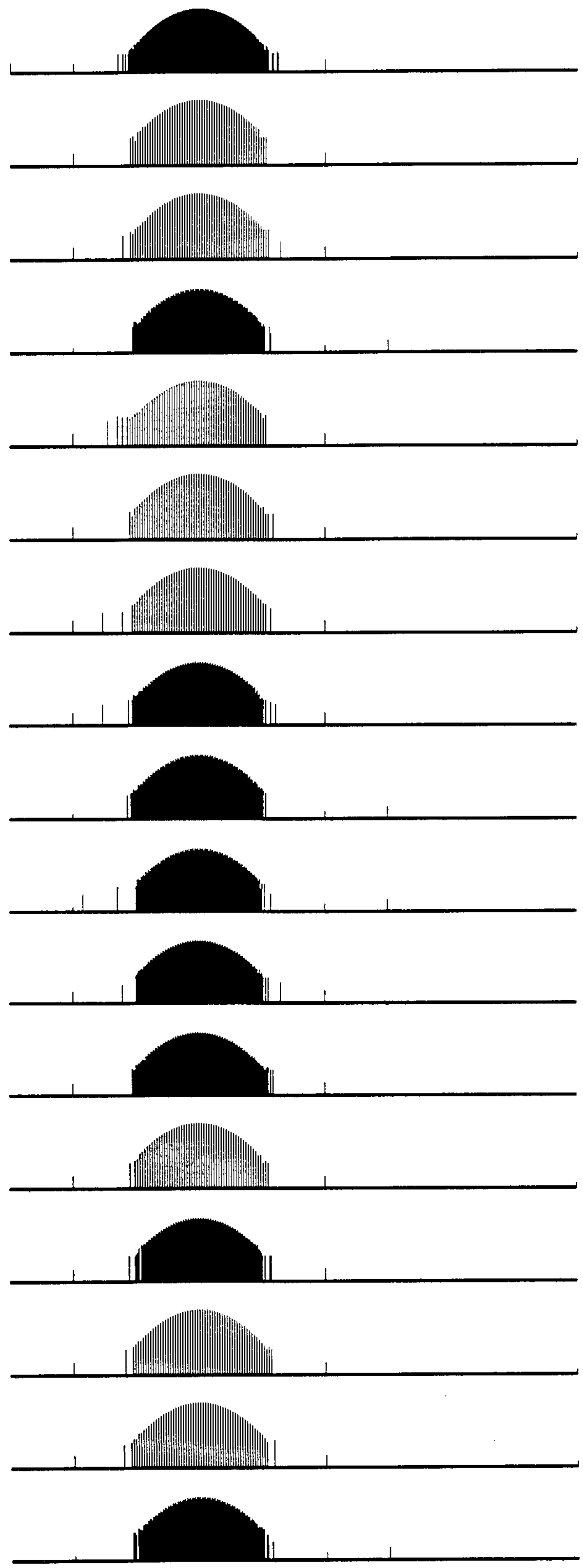

weights 714-1632
spectrum 101-104

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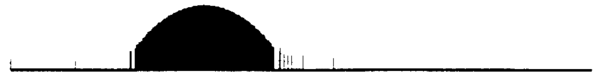

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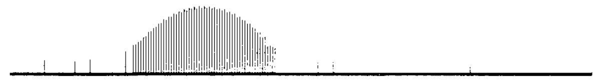

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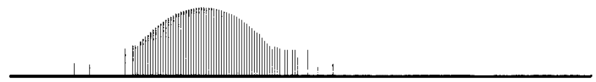

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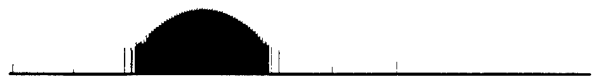

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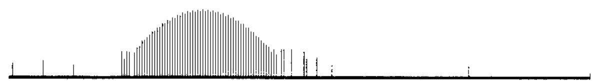

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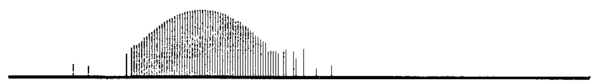

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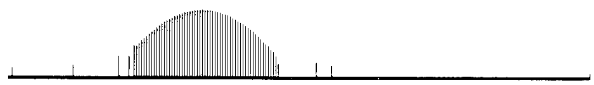

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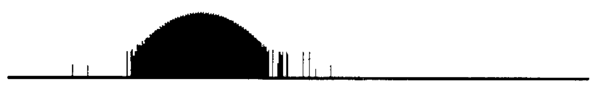

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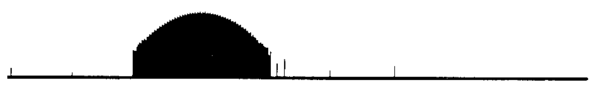

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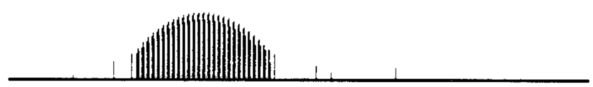

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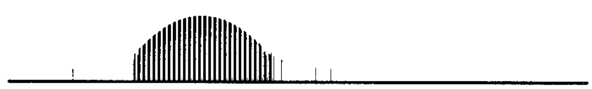

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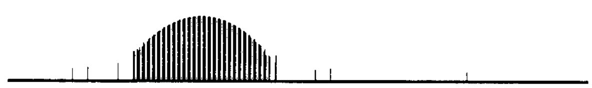

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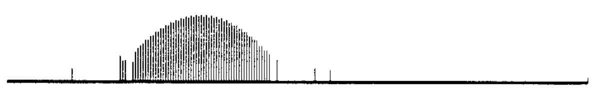

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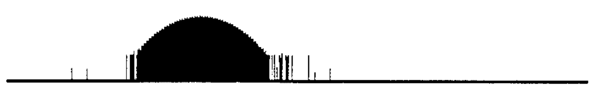

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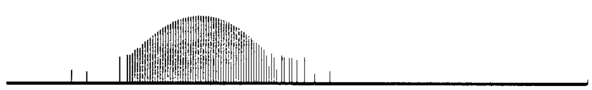

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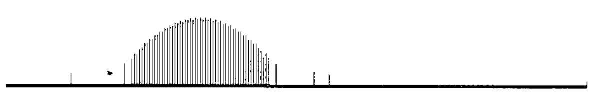

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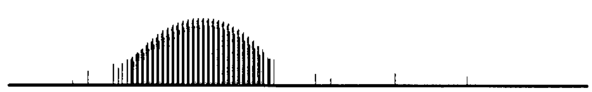

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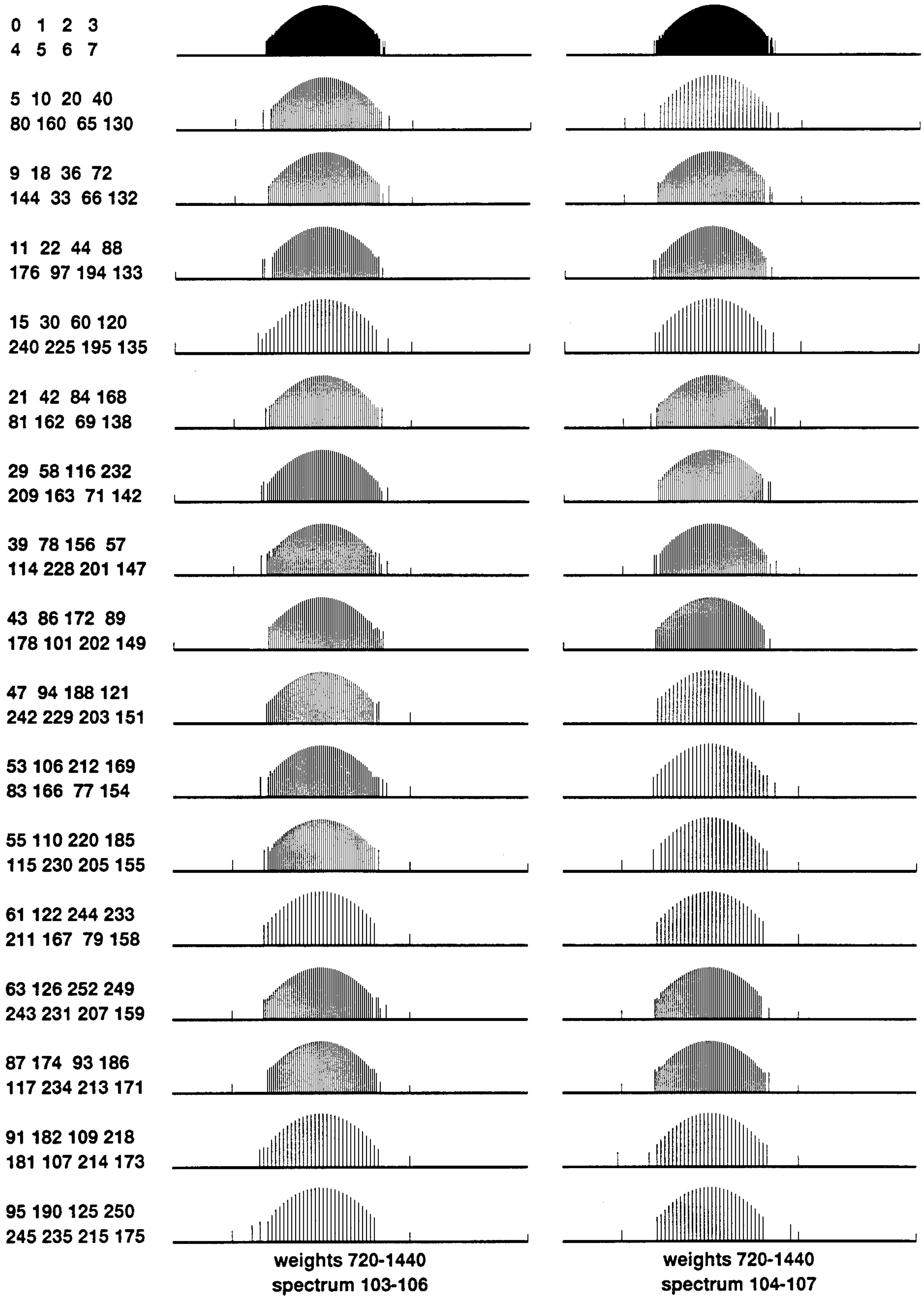
spectrum 102-105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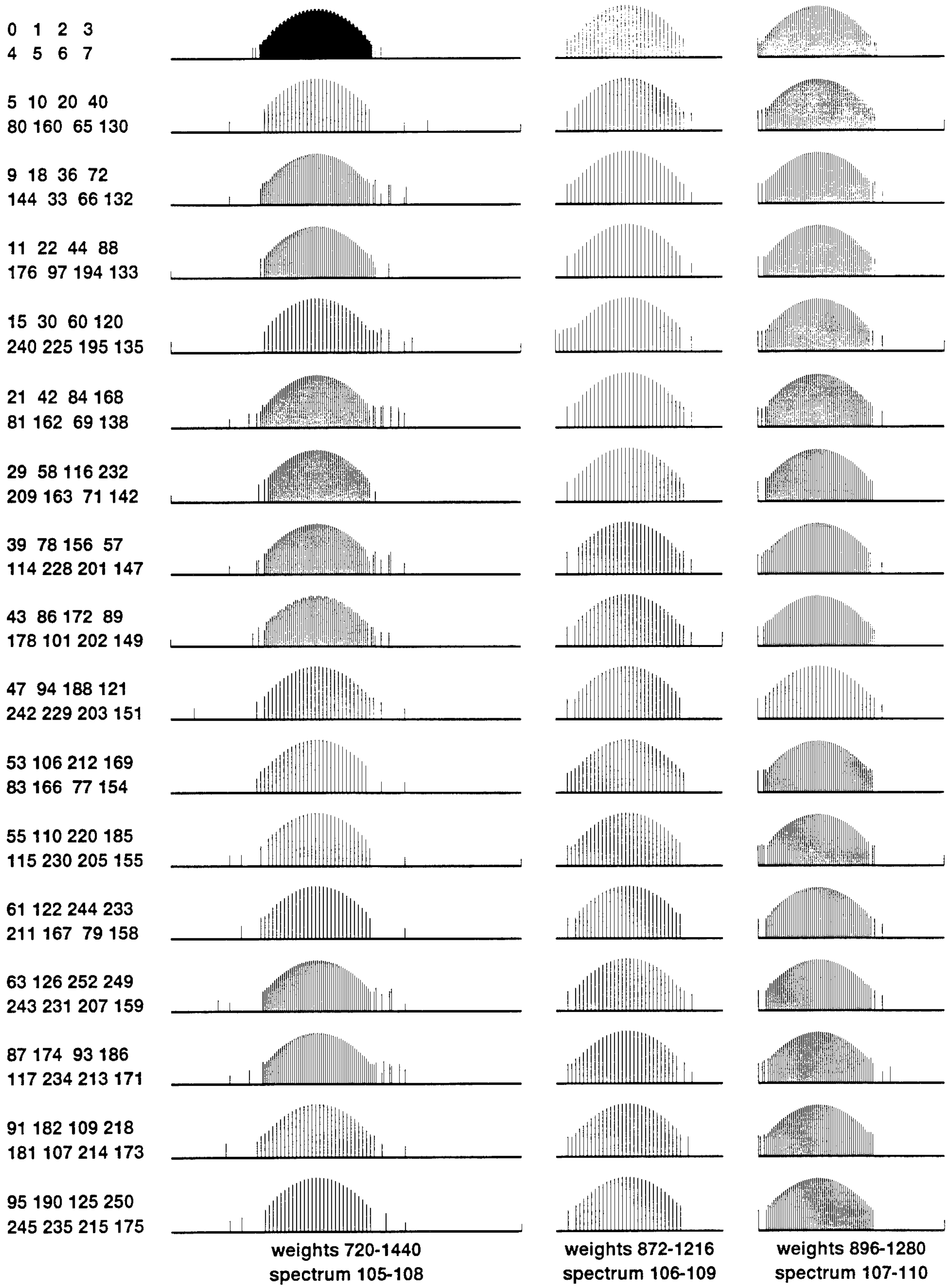
spectrum 103-106
weights 720-1440
spectrum 104-107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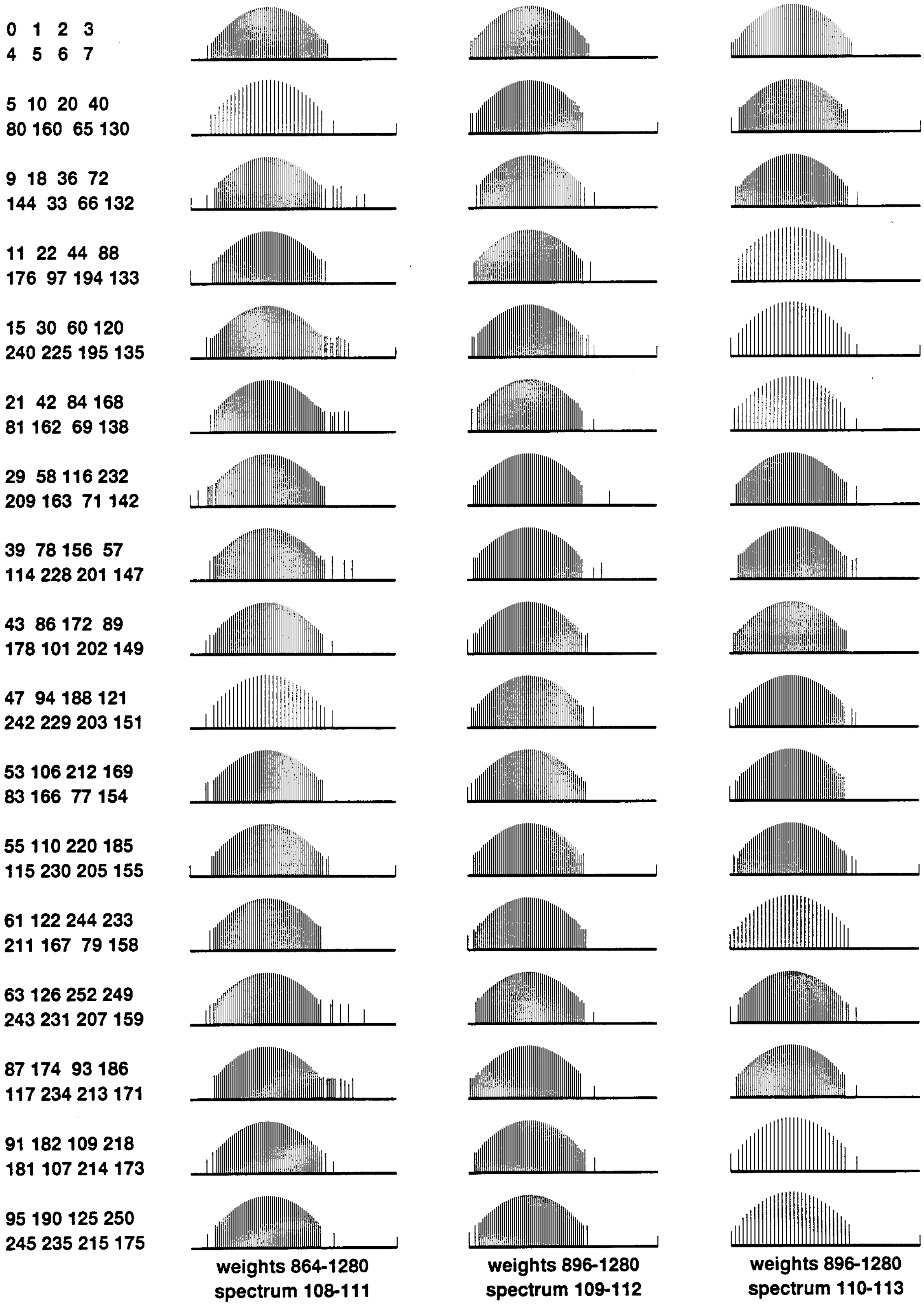
weights 872-1216
spectrum 106-109
weights 896-1280
spectrum 107-110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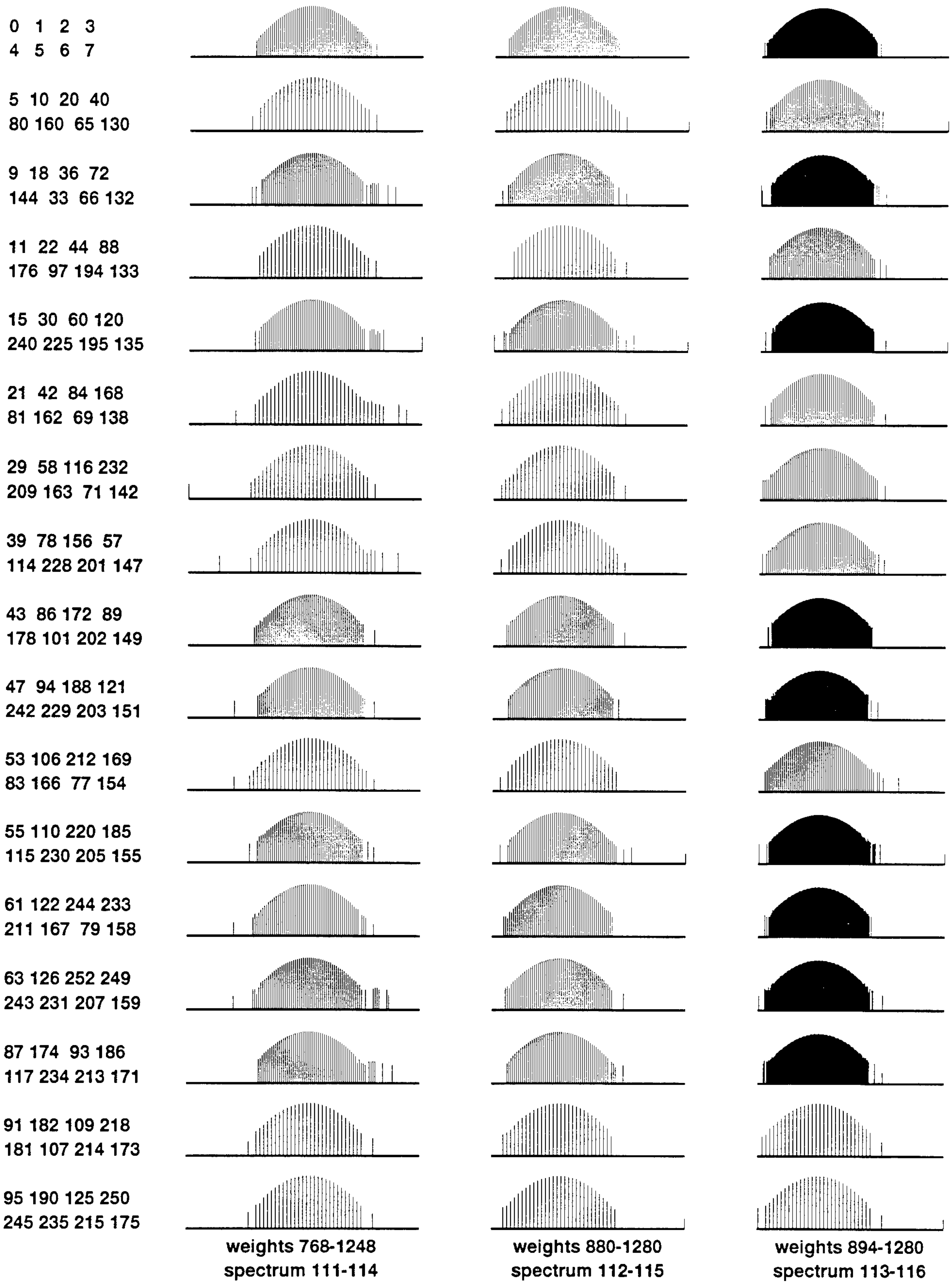
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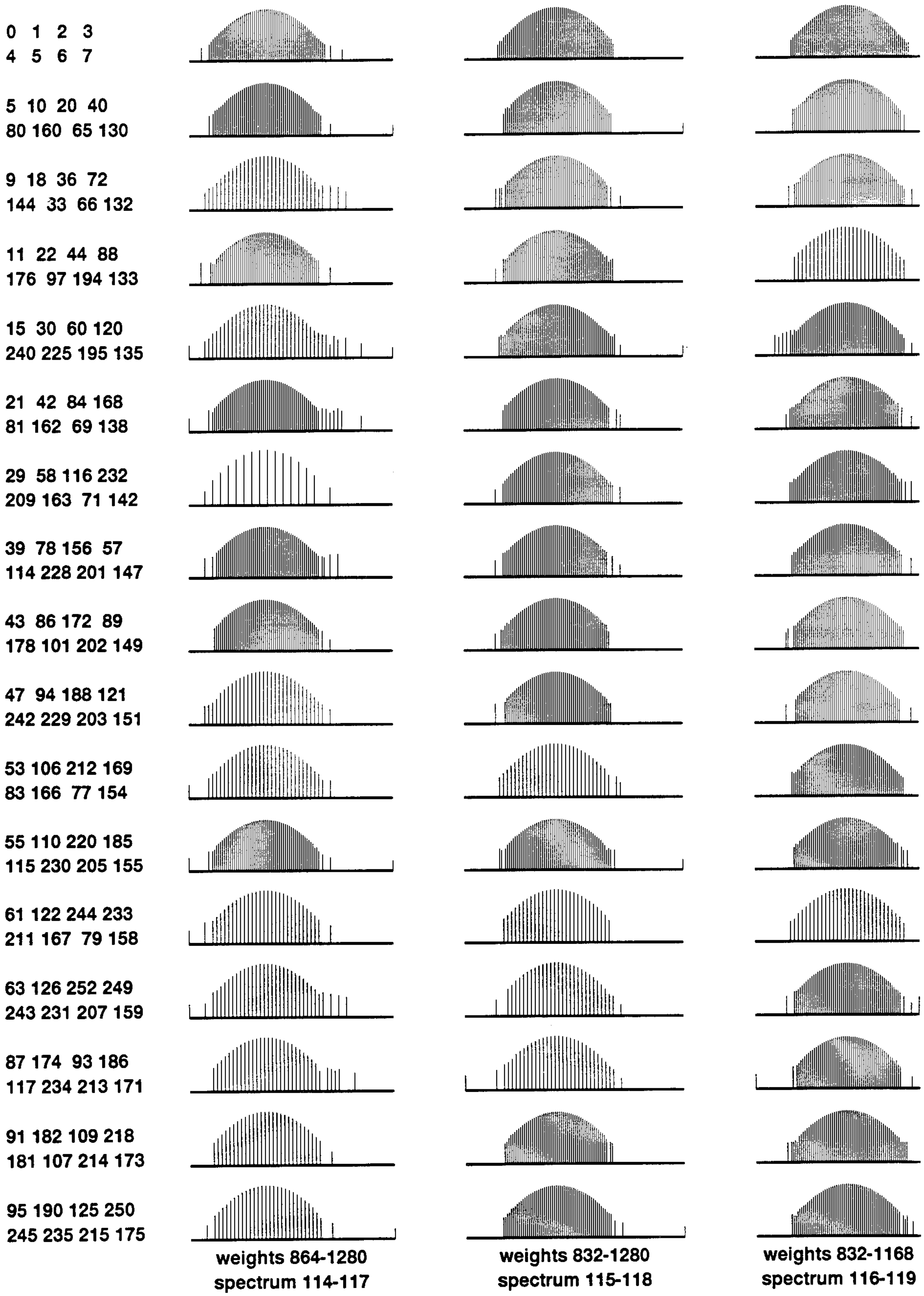
spectrum 111-114
weights 880-1280
spectrum 112-115
weights 894-1280
spectrum 113-116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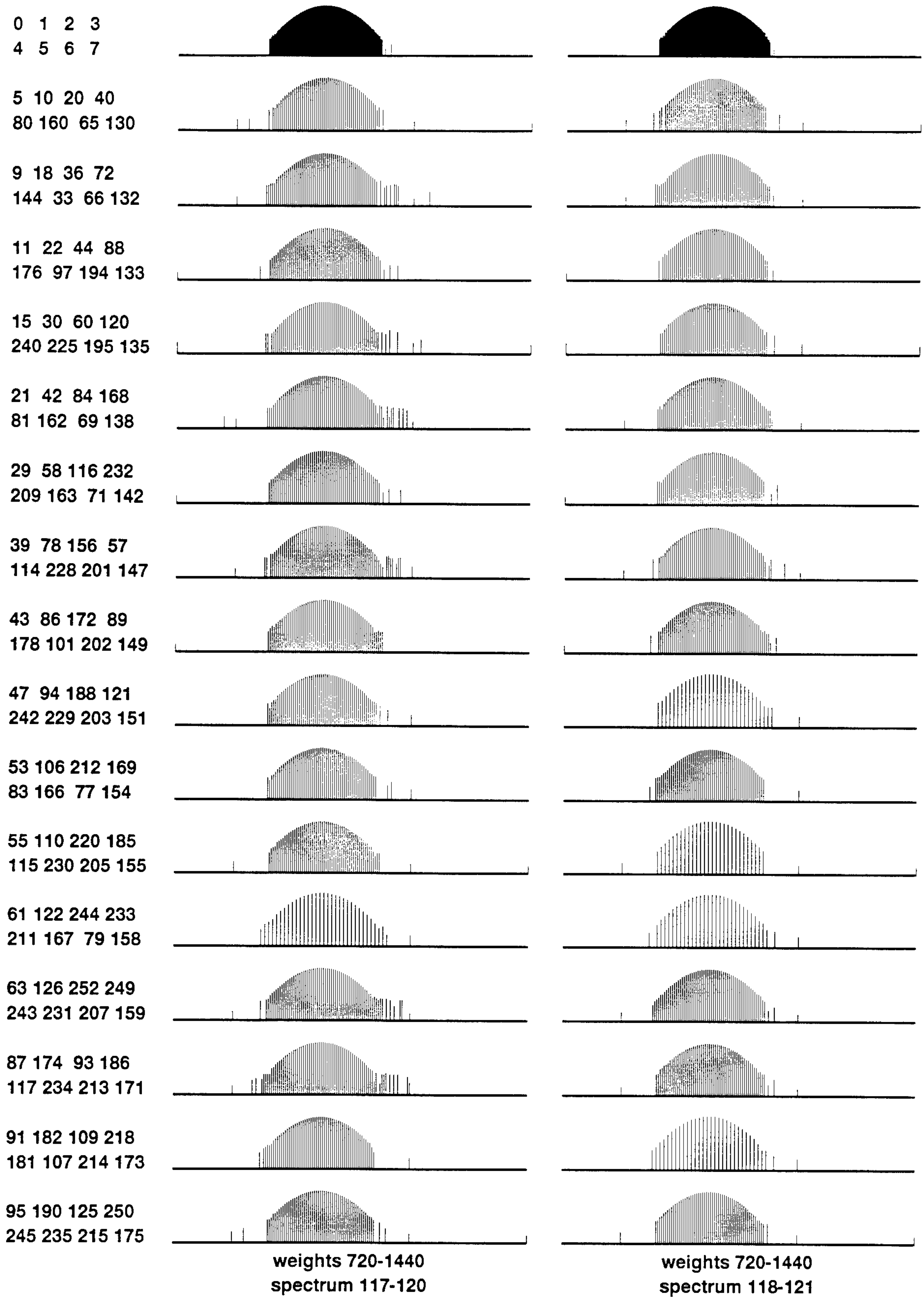
spectrum 114-117
weights 832-1280
spectrum 115-118
weights 832-1168
spectrum 116-119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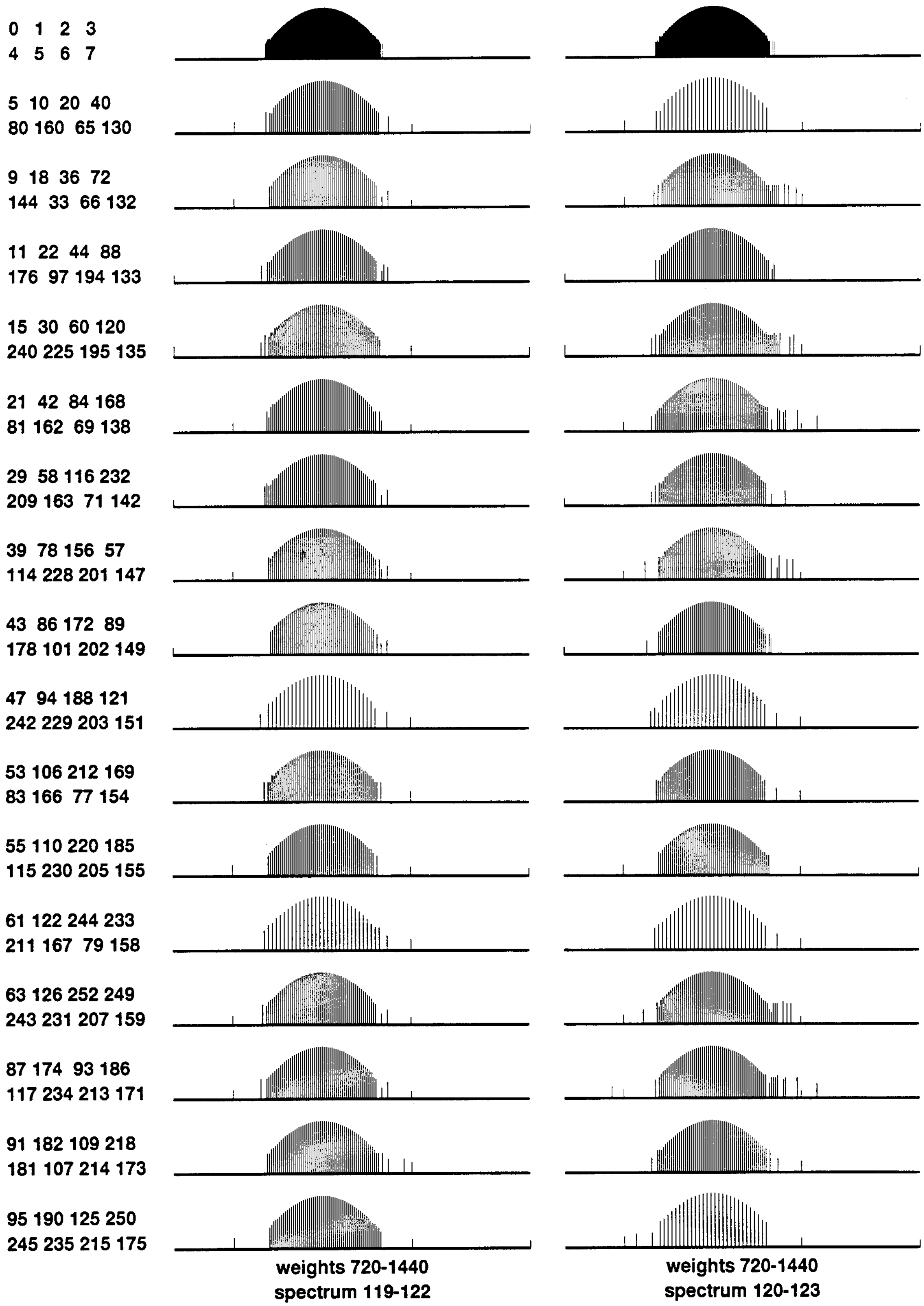
spectrum 117-120
weights 720-1440
spectrum 118-121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weights 720-1440
spectrum 119-122
weights 720-1440
spectrum 120-123

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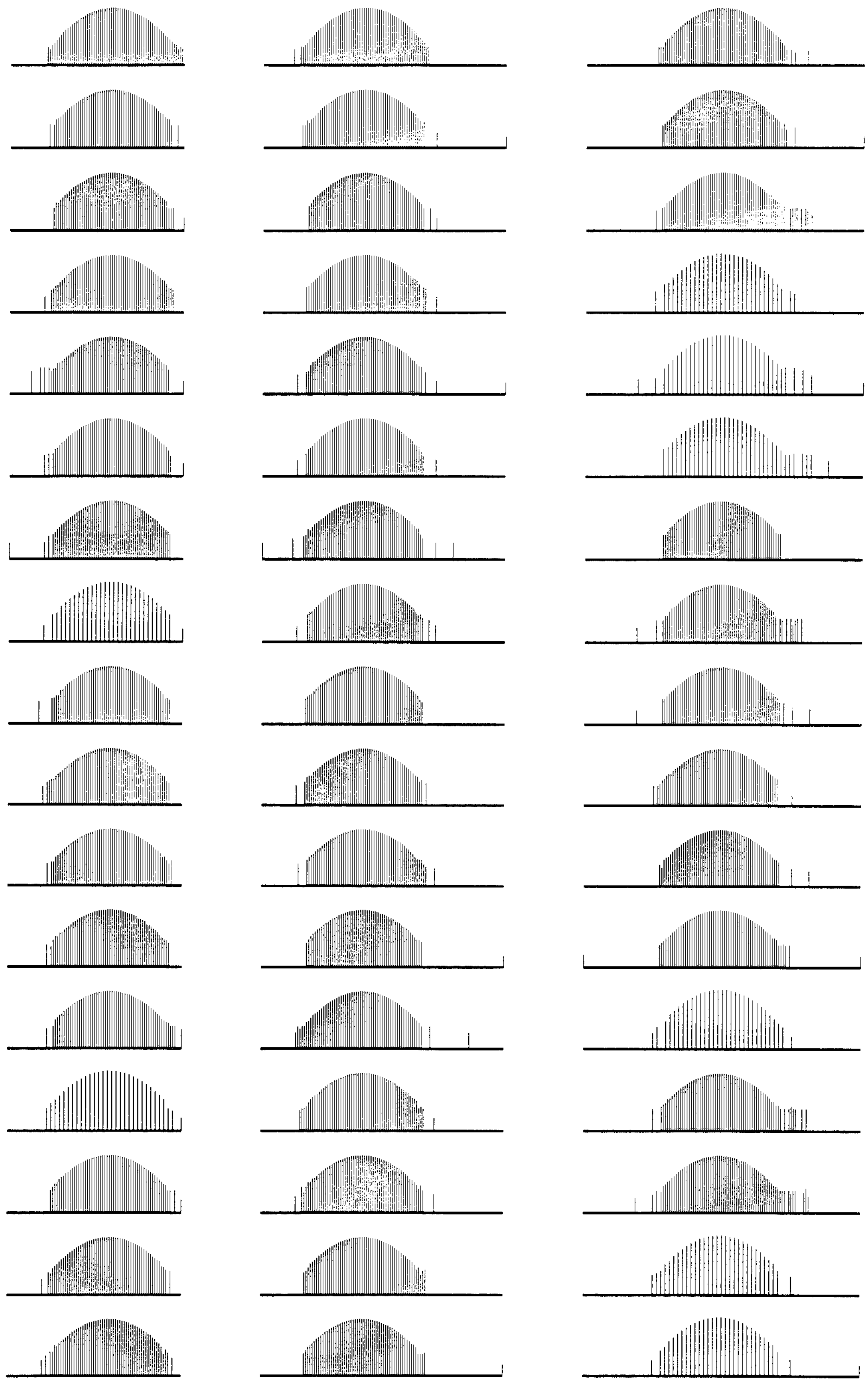

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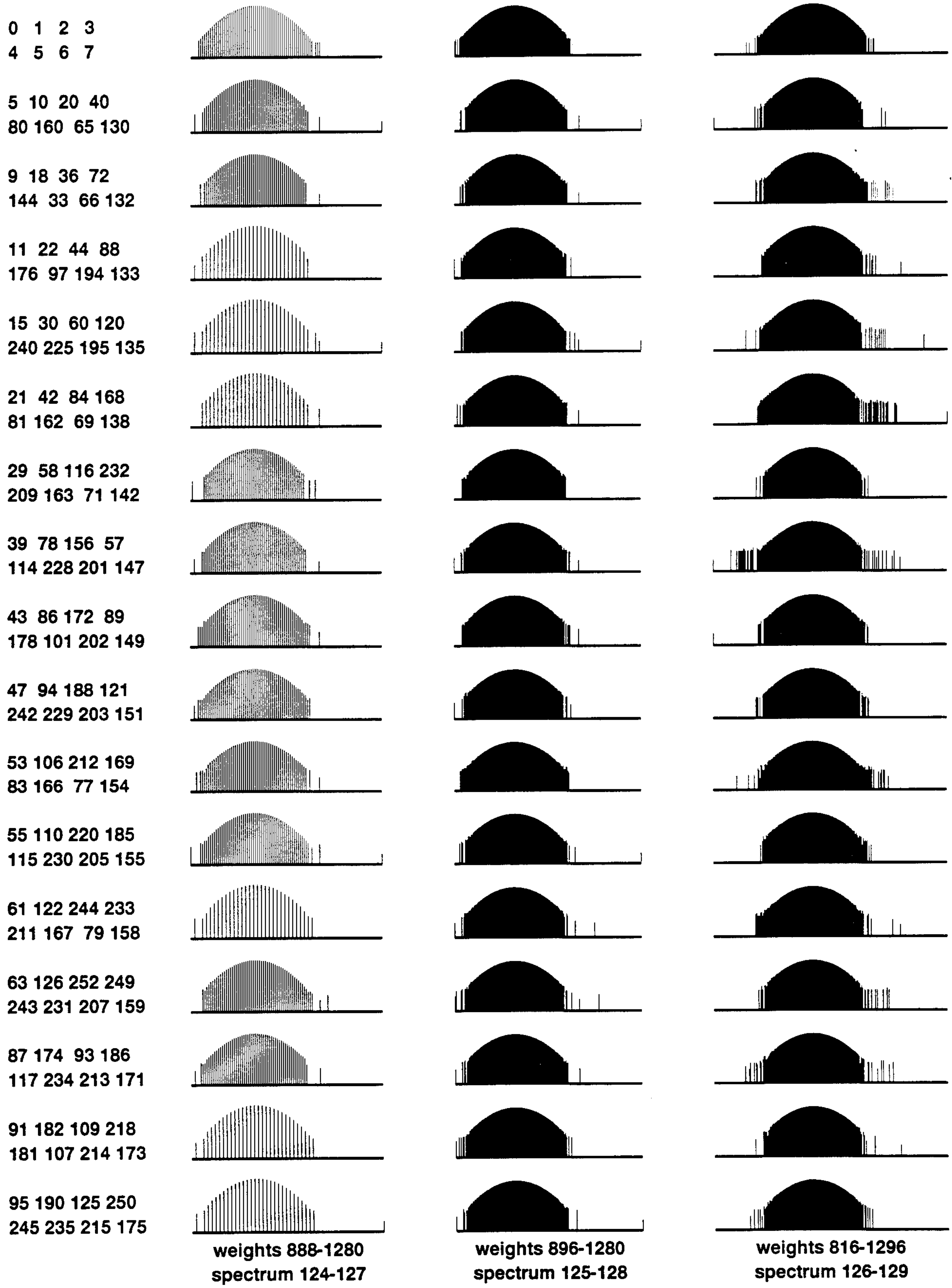
weights 832-1280
spectrum 122-125

weights 768-1280
spectrum 123-126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245 235 215 175
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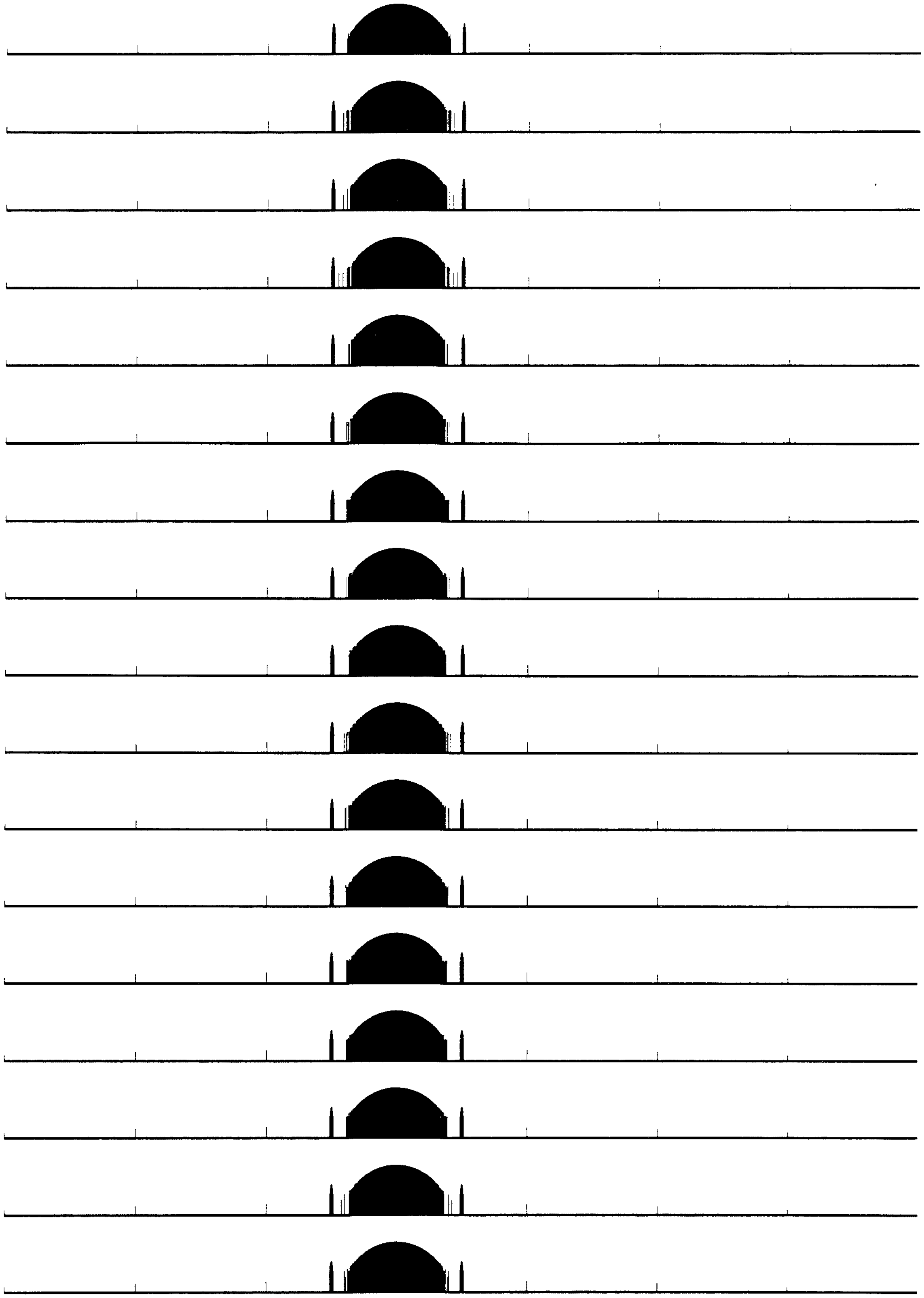
spectrum 124-127
weights 896-1280
spectrum 125-128
weights 816-1296
spectrum 126-129



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29 58 116 232
209 163 71 142

39 78 156 57
114 228 201 147

43 86 172 89
178 101 202 149

47 94 188 121
242 229 203 151

53 106 212 169
83 166 77 154

55 110 220 185
115 230 205 155

61 122 244 233
211 167 79 158

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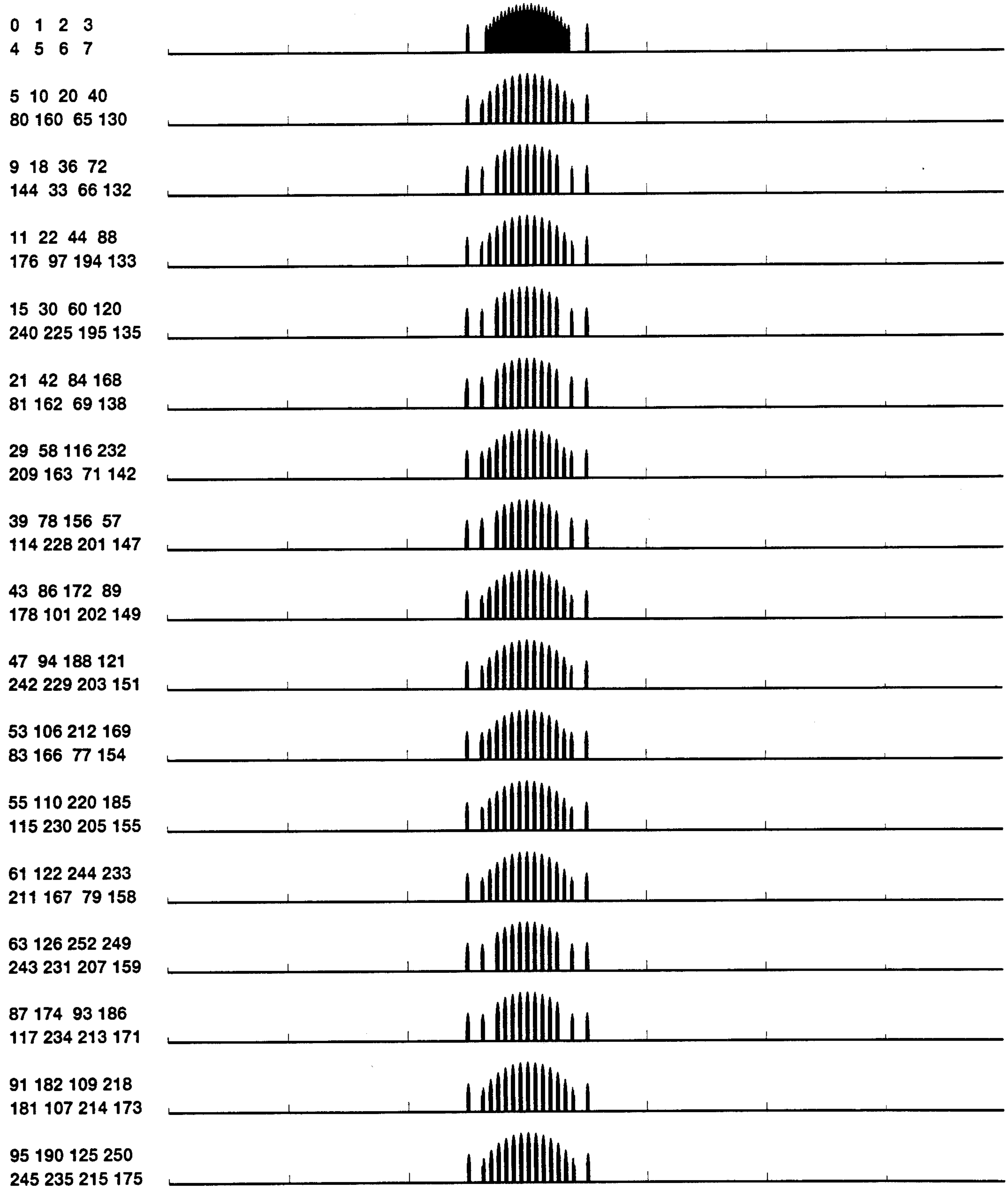
95 190 125 250
245 235 215 175

weights 255-2040
spectrum 254-2



weights 255-2040
spectrum 0-3

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